Membrane-Based Extraction Joined with Membrane-Based Stripping in a Circulating Arrangement

I. Modelling of Mass Transfer in a Hollow Fibre Contactor

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A simple model is developed for the determination of the overall coefficients of extraction and stripping, K_e , K_s , in membrane-based extraction joined with membrane-based stripping (reextraction) in circulating arrangement. Ideally mixed phases in the reservoirs are demanded. The calculation of mass transfer in hollow fibre contactors was based on integral equations for steady-state conditions. The equations of mass balance in the reservoirs (ordinary differential equations) together with equations for the calculation of exit concentrations of the transferred component in the phases flowing from the contactors (integral equations) formed a set of model equations. The overall coefficients are calculated from the time dependences of concentrations in both the feed and the solvent, or the stripping reservoirs. Constant flows of phases through hollow fibre contactors and constant volumes of phases in the reservoirs are supposed. The nonlinear dependence of partition coefficients, D_F , D_R , on concentration of the transferred component can be used.

The determination of the overall extraction coefficients for a noncirculating arrangement [1] of membrane-based extraction or membrane-based stripping requires a big volume of phases, because the flow of phases through hollow fibre contactor (HFC) is high already for a quite small contactor. In this case, however, experiments are expensive and often the laboratory equipment has to be adapted.

The circulating arrangement of the membranebased extraction for the determination of the overall extraction coefficients is an adequate solution of this problem. In this arrangement, the feed phase is returning back to the feed solution reservoir after passing HFC usually through fibres. Similarly is arranged the flow of solvent. The solvent passes HFC usually through its shell side and returns back to the reservoir of the solvent. Membrane-based extraction joined with simultaneous membrane-based stripping is the case, which is very often used. In this arrangement, after passing extraction HFC the solvent flows through the shell side of the stripping HFC, and then it returns back to the reservoir of the solvent. After passing the stripping HFC through fibres stripping solution returns back to the reservoir of the stripping solution.

THEORETICAL

At steady-state conditions in a noncirculating arrangement of the phase flow the overall extraction coefficient, K_{e} , in HFC is determined from the concen-

trations at the inlet and outlet of the module. In determining the overall extraction coefficient, $K_{\rm e}$, in HFC by the model the following assumptions were used:

- a) Steady state (no change in temperature and concentration fields inside of the module) resulting from constant flows and constant concentrations at the inlet and outlet of the module.
- b) Plug flow of fluid inside of the hollow fibres with the same flow in all fibres.
 - c) Plug flow of fluid in the shell side of the module.
 - d) Radial concentration profiles do not exist.
- e) Axial concentration profile arises as a consequence of mass transfer of the component.
 - f) Axial diffusion is negligible.
- g) Concentration changes do not influence the flow rate. The flow rates of feed (aqueous) and solvent (organic) phases, $\dot{V}_{\rm F}$ and $\dot{V}_{\rm S}$, are constant.
- h) The partition coefficient $D_{\rm F}$ of extraction is constant.
 - i) The interfacial area $A_{\rm e}$ is known.
- j) Reactions on the phase interfaces are sufficiently rapid and only mass transfer resistances are considered.

At the countercurrent arrangement of the flow streams in the module the balance of the extracted component for the whole contactor can be expressed as follows

$$\dot{V}_{\rm F} (c_{\rm F1} - c_{\rm F2}) = \dot{V}_{\rm S} (c_{\rm S1} - c_{\rm S0})$$
 (1)

The balances of the extracted component for a part of

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the contactor are expressed as

$$\dot{V}_{\rm F}(c_{\rm F1} - c_{\rm F}) = \dot{V}_{\rm S}(c_{\rm S1} - c_{\rm S})$$
 (2)

$$\dot{V}_{\rm F}(c_{\rm F} - c_{\rm F2}) = \dot{V}_{\rm S}(c_{\rm S} - c_{\rm S0})$$
 (3)

For any cross-section of the hollow fibre contactor, the rate of mass transfer of the extracted component is proportional to the concentration driving force

$$-w\left(\frac{\pi}{4}d_{\rm i}^2N\right)\frac{\mathrm{d}c_{\rm F}}{\mathrm{d}z} = K_{\rm e}\left(\pi d_{\rm i}N\varepsilon\right)\left(c_{\rm F} - \frac{c_{\rm S}}{D_{\rm F}}\right) \quad (4)$$

where w is the average velocity of the fluid in each fibre (aqueous phase), d_i the inner diameter of the hollow fibre, z the length coordinate, ε the porosity of the fibre wall, and N the number of hollow fibres in the contactor.

By modifying eqn (4), combining with eqns (1) and (2) and subsequent integration, the following expression is obtained

$$\ln \frac{c_{\rm F2} - \frac{c_{\rm S0}}{D_{\rm F}}}{c_{\rm F1} - \frac{c_{\rm S1}}{D_{\rm F}}} = -(1 - E_{\rm F}) N_{\rm F}$$
 (5)

where $N_{\rm F}$ is the number of transfer units expressed as

$$N_{\rm F} = \frac{K_{\rm e}A_{\rm e}}{\dot{V}_{\rm F}} \tag{6}$$

in which $A_{\rm e}$ defined as

$$A_{\rm e} = NL\pi d_{\rm i}\varepsilon\tag{7}$$

is the actual interface area of the extraction module and $E_{\rm F}$ (inverse quantity of the extraction factor) is given by the following relation

$$E_{\rm F} = \frac{\dot{V}_{\rm F}}{D_{\rm E}\dot{V}_{\rm S}} \tag{8}$$

For the known value of the overall coefficient, $K_{\rm e}$, relation (5) can be modified to a form enabling the calculation of the outlet concentration $c_{\rm F2}$ from the inlet concentrations $c_{\rm F1}$ and $c_{\rm S0}$

$$c_{\rm F2} = c_{\rm F1} \frac{(1 - E_{\rm F}) e^{-W}}{1 - E_{\rm F} e^{-W}} + c_{\rm S0} \frac{1 - e^{-W}}{D_{\rm F} (1 - E_{\rm F} e^{-W})}$$
(9)

where

$$W = (1 - E_{\rm F}) N_{\rm F} \tag{10}$$

For the cocurrent flow of phases in the module the following relation is defined by a similar procedure as for the countercurrent flow of phases

$$\ln \frac{c_{\rm F2} - \frac{c_{\rm S1}}{D_{\rm F}}}{c_{\rm F1} - \frac{c_{\rm S0}}{D_{\rm F}}} = -(1 + E_{\rm F}) N_{\rm F}$$
 (11)

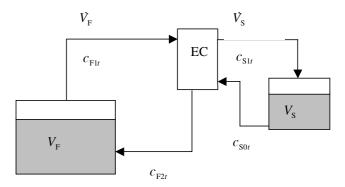


Fig. 1. Scheme of the circulating arrangement of extraction. $V_{\rm F}$ – volume of the feed phase, $V_{\rm S}$ – volume of the solvent phase, EC – extraction contactor.

For the calculation of the outlet concentration of feed $c_{\rm F2}$ from the inlet concentrations $c_{\rm F1}$ and $c_{\rm S0}$ and a known value of $K_{\rm e}$ the following relation is used

$$c_{\rm F2} = c_{\rm F1} \frac{E_{\rm F} + e^{-O}}{1 + E_{\rm F}} + c_{\rm S0} \frac{1 - e^{-O}}{D_{\rm F} (1 + E_{\rm F})}$$
 (12)

where

$$O = N_{\rm F} (1 + E_{\rm F})$$
 (13)

A method of determination of the overall extraction coefficient, $K_{\rm e}$, in HFC for a circulating arrangement of extraction from the time dependence of concentrations of the extracted component in an ideally mixed reservoir was proposed.

The scheme of the circulating arrangement of extraction is given in Fig. 1.

The balance of the extracted component in the feed reservoir, in differential time periods for $0 < t < t_{\rm end}$ can be expressed as

$$-V_{\rm F} \frac{\mathrm{d}c_{\rm F1t}}{\mathrm{d}t} = \dot{V}_{\rm F} \left(c_{\rm F1t} - c_{\rm F2t} \right) \tag{14}$$

For the countercurrent arrangement of the flow through HFC, the following form of balance equation is obtained by substituting eqn (9) into eqn (14) and its rearranging

$$-\frac{\mathrm{d}c_{\mathrm{F}1t}}{\mathrm{d}t} = \frac{1 - \mathrm{e}^{-W}}{\tau_{\mathrm{F}} (1 - E_{\mathrm{F}} \mathrm{e}^{-W})} \left(c_{\mathrm{F}1t} - \frac{c_{\mathrm{S}0t}}{D_{\mathrm{F}}} \right) \tag{15}$$

where $\tau_{\rm F}$ is

$$\tau_{\rm F} = \frac{V_{\rm F}}{\dot{V}_{\rm F}} \tag{16}$$

For the cocurrent arrangement of the flow through HFC the following expression was formulated

$$-\frac{\mathrm{d}c_{\mathrm{F}1t}}{\mathrm{d}t} = \frac{1 - \mathrm{e}^{-O}}{\tau_{\mathrm{F}} (1 + E_{\mathrm{F}})} \left(c_{\mathrm{F}1t} - \frac{c_{\mathrm{S}0t}}{D_{\mathrm{F}}} \right) \tag{17}$$

The balance of the extracted component in the whole extraction apparatus, from the beginning of extraction to arbitrary time of extraction, is described as follows

$$V_{\rm F}(c_{\rm F0} - c_{\rm F1}t) = V_{\rm S}(c_{\rm S0}t - c_{\rm S0})$$
 (18)

For t = 0 usually $c_{S0} = 0$. Then

$$c_{S0t} = c_{F0} \frac{V_F}{V_S} - c_{F1t} \frac{V_F}{V_S}$$
 (19)

By substituting eqn (19) into eqn (15) and rearranging one gets

$$-\frac{\mathrm{d}c_{\mathrm{F}1t}}{\mathrm{d}t} = \frac{1}{\tau_{\mathrm{F}}} [X c_{\mathrm{F}1t} - Y] \tag{20}$$

where

$$X = \frac{(1 + \bar{E}_{\rm F}) (1 - e^{-W})}{1 - E_{\rm F} e^{-W}}$$
 (21)

$$Y = \frac{\bar{E}_{\rm F} \left(1 - e^{-W} \right)}{1 - E_{\rm F} e^{-W}} c_{\rm F0} \tag{22}$$

in which quantities X and Y are constant values when conditions g(t), t(t), t(t), and t(t) are fulfilled and

$$\bar{E}_{\rm F} = \frac{V_{\rm F}}{D_{\rm F} V_{\rm S}} \tag{23}$$

The integral of eqn (20) gives the following form

$$C_{\text{F1}t} = \frac{c_{\text{F1}t}}{c_{\text{F0}}} = \frac{\bar{E}_{\text{F}}}{1 + \bar{E}_{\text{F}}} + \frac{1}{1 + \bar{E}_{\text{F}}} \exp(-X\Theta)$$
 (24)

where the dimensionless time is defined as

$$\Theta = \frac{t}{\tau_{\rm F}} \tag{25}$$

For the cocurrent flow of phases through the contactor the following expression was derived

$$C_{\text{F}1t} = \frac{c_{\text{F}1t}}{c_{\text{F}0}} = \frac{\bar{E}_{\text{F}}}{1 + \bar{E}_{\text{F}}} + \frac{1}{1 + \bar{E}_{\text{F}}} \exp(-Z\Theta)$$
 (26)

 $_{
m where}$

$$Z = \frac{1 - (1 - \bar{E}_{\rm F}) e^{-O}}{1 + E_{\rm F}}$$
 (27)

For the determination of the overall coefficient, K_e , eqn (24) is transformed to the following form

$$\ln \frac{c_{F1t} - c_{F0} \frac{\bar{E}_F}{1 + \bar{E}_F}}{\frac{c_{F0}}{1 + \bar{E}_F}} = -X\Theta$$
 (28)

The overall coefficient, $K_{\rm e}$, is determined from the experimental time dependence of the concentration of the extraction component in the feed reservoir. The plot of experimental values calculated according to relation (28) has the shape of a straight line and $K_{\rm e}$ is determined from the slope of this straight line. An analogous procedure is used for the evaluation of the overall coefficient $K_{\rm e}$ from relation (26).

The relations given in the previous chapters were presented in similar forms by several other authors [2—11].

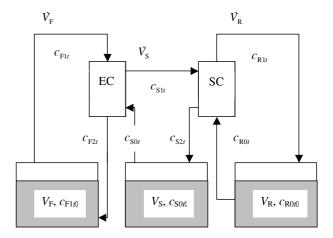


Fig. 2. Scheme of the circulating arrangement of extraction joined with stripping (reextraction). EC, SC – extraction and stripping contactors, $V_{\rm F}$, $V_{\rm S}$, $V_{\rm R}$ – volume of the feed, solvent, and stripping phases. $\dot{V}_{\rm F}$, $\dot{V}_{\rm S}$, $\dot{V}_{\rm R}$ – flow rates of the feed, solvent, and stripping phases, $c_{\rm F}$, $c_{\rm S}$, $c_{\rm R}$ – concentrations of the transferred component

RESULTS AND DISCUSSION

The circulating arrangement of extraction joined with stripping (reextraction) is schematically drawn in Fig. 2. This arrangement was considered as an arrangement for two consecutive and simultaneous extractions: extraction of the transferred component from the feed (raffinate) phase into the solvent phase and extraction of this component from the solvent phase into the stripping phase.

For the purpose of modelling this process, the aim of which was determination of the extraction coefficient, $K_{\rm e}$, and the reextraction (stripping) coefficient, $K_{\rm s}$, in HFC, the overall scheme is divided into three individual schemes, each one corresponding to the pertinent phase, as shown in Fig. 3.

Feed-Raffinate Loop

Because simultaneously with extraction of the extracted component from the feed (raffinate) to the solvent, stripping of this component from the solvent to the stripping phase proceeds, eqn (18) is not valid.

For the mathematical description of the model, eqn (15) for countercurrent flow and eqn (17) for cocurrent flow of phases through HFC were applied.

Note: Eqn (9) used for rearranging of eqn (14) represented one passing of the fluid through the contactor. The concentration change caused by passing the fluid through the extraction contactor is small. Thus, a constant distribution coefficient, $D_{\rm F}$, could be considered. Assumption a) can be still considered as valid, because a change of the concentration of the transferred component in the reservoir with time is sufficiently slow. Therefore, an approximately steady state in the extraction contactor could be assumed.

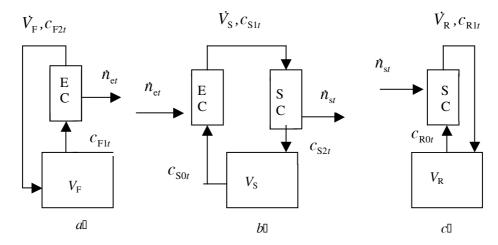


Fig. 3. Scheme for modelling the process of extraction with reextraction in the circulating arrangement. a – feed-raffinate loop, b – solvent loop, c – stripping phase loop.

The numerical solution of differential eqns (15) and (17) allows using the dependence of the partition coefficient on concentration of the transferred component in the solvent phase (assumption h) could be omitted)

$$D_{\rm F} = f\left(c_{\rm S}\right) \tag{29}$$

Concentrations c_{S0t} in eqns (15) and (17) are calculated from pertinent equations concerning the solvent loop (see the next part).

Solvent Loop

The flow of the extracted component from the feed phase into the solvent phase in the extraction contactor with negligible volume can be expressed as follows

$$\dot{n}_{et} = \dot{V}_{F} \left(c_{F1t} - c_{F2t} \right) \tag{30}$$

Similarly, for the mass flow of the extracted component from the solvent phase into the stripping phase in the stripping contactor, the balance of the extracted component can be described as follows

$$\dot{n}_{\rm st} = \dot{V}_{\rm S} \left(c_{\rm S1t} - c_{\rm S2t} \right)$$
 (31)

For concentrations $c_{\mathrm{S1}t}$ and $c_{\mathrm{S2}t}$, the balance of the extracted component through the interfacial area in extraction and stripping modules for countercurrent arrangement of the flow through HFC leads to the relations

$$c_{S1t} = c_{S0t} + \frac{\dot{V}_F}{\dot{V}_S} (c_{F1} - c_{F2})$$
 (32)

$$c_{\rm S2} = c_{\rm S1} \frac{(1 - E_{\rm S}) \,\mathrm{e}^{-S}}{1 - E_{\rm S} \mathrm{e}^{-S}} + c_{\rm R0} \frac{1 - \mathrm{e}^{-S}}{D_{\rm R} \,(1 - E_{\rm S} \mathrm{e}^{-S})}$$
 (33)

where

$$S = (1 - E_{\rm S}) N_{\rm S}$$
 (34)

$$E_{\rm S} = \frac{\dot{V}_{\rm S}}{D_{\rm R}\dot{V}_{\rm R}} \tag{35}$$

$$N_{\rm S} = \frac{K_{\rm s}A_{\rm s}}{\dot{V}_{\rm S}} \tag{36}$$

and for the cocurrent arrangement of the flow through HFC the following relation was obtained

$$c_{S2} = c_{S1} \frac{E_S + e^{-U}}{1 + E_S} + c_{R0} \frac{1 - e^{-U}}{D_R (1 + E_S)}$$
(37)

where

$$U = N_{\rm S} (1 + E_{\rm S}) \tag{38}$$

The balance of the extracted component of the solvent reservoir is done for a differential time period for $0 < t < t_{\rm end}$ and takes the form

$$-V_{\rm S} \frac{\mathrm{d}c_{\rm S0t}}{\mathrm{d}t} = \dot{V}_{\rm S} \left(c_{\rm S0t} - c_{\rm S2t} \right) \tag{39}$$

After substituting eqn (32) and eqn (33) into eqn (39) and rearranging the exe form of balance for counter-current flow is obtained

$$-\frac{\mathrm{d}c_{S0t}}{\mathrm{d}t} = \tau_{S}^{-1} \left\{ c_{S0t} \frac{1 - \mathrm{e}^{-S}}{1 - E_{S}\mathrm{e}^{-S}} - \frac{\dot{V}_{F}}{\dot{V}_{S}} \left(c_{F1t} - c_{F2t} \right) \times \left(\frac{1 - E_{S}}{1 - E_{S}\mathrm{e}^{-S}} - c_{R0t} \frac{1 - \mathrm{e}^{-S}}{D_{R} \left(1 - E_{S}\mathrm{e}^{-S} \right)} \right\}$$
(40)

For cocurrent flow the following exe form was defined

$$-\frac{\mathrm{d}c_{S0t}}{\mathrm{d}t} = \tau_{S}^{-1} \left\{ c_{S0t} \frac{1 - \mathrm{e}^{-U}}{1 + E_{S}} - \frac{\dot{V}_{F}}{\dot{V}_{S}} \left(c_{F1t} - c_{F2t} \right) \times \frac{E_{S} + \mathrm{e}^{-U}}{1 + E_{S}} - c_{R0t} \frac{1 - \mathrm{e}^{-U}}{D_{R} \left(1 + E_{S} \right)} \right\}$$
(41)

where

$$\tau_{\rm S} = \frac{V_{\rm S}}{\dot{V}_{\rm S}} \tag{42}$$

Values of c_{S0t} calculated from eqns (40) or (41) are necessary for solving eqns (15) or (17). For numerical

solving of eqns (40) and (41) an initial value of c_{S0t} has to be known (for t = 0 c_{S0t} is usually zero).

In eqns (33), (35), (40), and (41) $D_{\rm R}$ is supposed to be a known function of the concentration of the stripping phase

$$D_{\rm R} = f\left(c_{\rm R}\right) \tag{43}$$

The concentration c_{R0t} in eqns (40) or (41) is calculated with pertinent equations given in the next part.

Note: For equations used in this part analogical conclusions were applied as in the note given in the section "Feed-Raffinate Loop" concerning the concentration change of the transferred component in the solute in extraction and stripping contactors.

Stripping Phase Loop

Supposing that the volumes of flowing phases inside of the module are negligibly small, the following balance of the stripped component in the stripping module is obtained

$$c_{R1t} = c_{R0t} + \frac{\dot{V}_{S}}{\dot{V}_{R}} (c_{S1t} - c_{S2t})$$
 (44)

Using eqn (44) and eqns (32), (33) the balance equation of the ideally mixed stripping reservoir with constant volume for the stripped component

$$-V_{\mathrm{R}}\frac{\mathrm{d}c_{\mathrm{R}0t}}{\mathrm{d}t} = \dot{V}_{\mathrm{R}}\left(c_{\mathrm{R}0t} - c_{\mathrm{R}1t}\right) \tag{45}$$

is transferred to the exe form (for countercurrent arrangement of the flow through HFC)

$$-\frac{\mathrm{d}c_{\mathrm{R}0t}}{\mathrm{d}t} = (\tau_{\mathrm{R}})^{-1} \frac{\dot{V}_{\mathrm{S}}}{\dot{V}_{\mathrm{R}}} \left\{ c_{\mathrm{R}0t} \frac{1 - \mathrm{e}^{-S}}{D_{\mathrm{R}} (1 - E_{\mathrm{S}} \mathrm{e}^{-S})} - \left[c_{\mathrm{S}0t} + \frac{\dot{V}_{\mathrm{F}}}{\dot{V}_{\mathrm{S}}} (c_{\mathrm{F}1t} - c_{\mathrm{F}2t}) \right] \frac{1 - \mathrm{e}^{-S}}{1 - E_{\mathrm{S}} \mathrm{e}^{-S}} \right\}$$
(46)

where

$$\tau_{\rm R} = \frac{V_{\rm R}}{\dot{V}_{\rm R}} \tag{47}$$

and for the cocurrent flow using eqn (44) and eqns (32), (37) the following exe relation was derived

$$-\frac{\mathrm{d}c_{\mathrm{R}0t}}{\mathrm{d}t} = (\tau_{\mathrm{R}})^{-1} \frac{\dot{V}_{\mathrm{S}}}{\dot{V}_{\mathrm{R}}} \left\{ c_{\mathrm{R}0t} \frac{1 - \mathrm{e}^{-U}}{D_{\mathrm{R}} (1 + E_{\mathrm{S}})} - \left[c_{\mathrm{S}0t} + \frac{\dot{V}_{\mathrm{F}}}{\dot{V}_{\mathrm{S}}} (c_{\mathrm{F}1t} - c_{\mathrm{F}2t}) \right] \frac{1 - \mathrm{e}^{-U}}{1 + E_{\mathrm{S}}} \right\}$$
(48)

Note: For equations used in this part analogical conclusions were applied as in the note in the section "Feed-Raffinate Loop" concerning the concentration change of the transferred component in the stripping phase in the stripping contactor.

CONCLUSION

A simple maximum gradient model with lumped parameters was proposed. Simplicity of the proposed model consists in the presumption that the mass transfer at unsteady state in the hollow fibre contactors can be sufficiently and exactly described by equations for mass transfer at steady conditions. In batch extraction joined with stripping, both extraction and stripping ran at unsteady conditions during the whole experiment. However, changes of the concentration of the transferred component are slow practically during the whole time of the experiment.

With model equations, a set of ordinary differential equations and integral equations can be easily handled. A nonlinear dependence of the distribution coefficient on the concentration of the transferred component in both, feed and solvent, phases can be used.

In addition, the proposed model assumes a simple experiment. In order to obtain sufficient experimental data the determination of the time dependence of the concentration of the transferred component in two reservoirs is sufficient. These reservoirs can be either feed and solvent reservoirs or feed and stripping reservoirs.

List of Equations for the Model of the Circulating Arrangement of Integrating Extraction with Reextraction

Auxiliary Equations of the Model

$$N_{\rm F} = \frac{K_{\rm e}A_{\rm e}}{\dot{V}_{\rm E}} \tag{6}$$

$$E_{\rm F} = \frac{\dot{V}_{\rm F}}{D_{\rm F}\dot{V}_{\rm S}} \tag{8}$$

$$W = (1 - E_{\rm F}) N_{\rm F}$$
 (10)

$$O = N_{\rm F} (1 + E_{\rm F})$$
 (13)

$$S = (1 - E_{\rm S}) N_{\rm S}$$
 (34)

$$E_{\rm S} = \frac{\dot{V}_{\rm S}}{D_{\rm R}\dot{V}_{\rm R}} \tag{35}$$

$$N_{\rm S} = \frac{K_{\rm s}A_{\rm s}}{\dot{V}_{\rm S}} \tag{36}$$

$$U = N_{\rm S} (1 + E_{\rm S})$$
 (38)

$$\tau_{\rm F} = \frac{V_{\rm F}}{\dot{V}_{\rm D}} \tag{16}$$

$$\tau_{\rm S} = \frac{V_{\rm S}}{\dot{V}_{\rm S}} \tag{42}$$

$$\tau_{\rm R} = \frac{V_{\rm R}}{\dot{V}_{\rm R}} \tag{47}$$

Governing Equations of the Model

Countercurrent arrangement of the flow of phases through HFC:

 $D_{\rm F} = f(c_{{\rm S0}t})$ (usually a polynomial form of the rela- $D_{\rm R} = f(c_{{\rm R}0t})$ (usually a polynomial form of the relation) (43)

$$-\frac{\mathrm{d}c_{\mathrm{F}1t}}{\mathrm{d}t} = \frac{1 - \mathrm{e}^{-O}}{\tau_{\mathrm{F}} (1 - E_{\mathrm{F}})} \left(c_{\mathrm{F}1t} - \frac{c_{\mathrm{S}0t}}{D_{\mathrm{F}}} \right) \tag{17}$$

$$c_{\rm F2} = c_{\rm F1} \frac{(1 - E_{\rm F}) e^{-W}}{1 - E_{\rm F} e^{-W}} + c_{\rm S0} \frac{1 - e^{-W}}{D_{\rm F} (1 - E_{\rm F} e^{-W})}$$
(9)

$$-\frac{\mathrm{d}c_{\mathrm{S}0t}}{\mathrm{d}t} = \tau_{\mathrm{S}}^{-1} \left\{ c_{\mathrm{S}0t} \frac{1 - \mathrm{e}^{-S}}{1 - E_{\mathrm{S}}\mathrm{e}^{-S}} - \frac{\dot{V}_{\mathrm{F}}}{\dot{V}_{\mathrm{S}}} \left(c_{\mathrm{F}1t} - c_{\mathrm{F}2t} \right) \times \left(\frac{(1 - E_{\mathrm{S}}) \,\mathrm{e}^{-S}}{1 - E_{\mathrm{S}}\mathrm{e}^{-S}} - c_{\mathrm{R}0t} \frac{1 - \mathrm{e}^{-S}}{D_{\mathrm{R}} \left(1 - E_{\mathrm{S}}\mathrm{e}^{-S} \right)} \right\}$$
(40)

$$c_{S1t} = c_{S0t} + \frac{\dot{V}_F}{\dot{V}_S} (c_{F1} - c_{F2})$$
 (32)

$$c_{S2} = c_{S1} \frac{(1 - E_S) e^{-S}}{1 - E_S e^{-S}} + c_{R0} \frac{1 - e^{-S}}{D_R (1 - E_S e^{-S})}$$
(33)

$$-\frac{\mathrm{d}c_{\mathrm{R}0t}}{\mathrm{d}t} = (\tau_{\mathrm{R}})^{-1} \frac{\dot{V}_{\mathrm{S}}}{\dot{V}_{\mathrm{R}}} \left\{ c_{\mathrm{R}0t} \frac{1 - \mathrm{e}^{-S}}{D_{\mathrm{R}} (1 - E_{\mathrm{S}} \mathrm{e}^{-S})} - \left[c_{\mathrm{S}0t} + \frac{\dot{V}_{\mathrm{F}}}{\dot{V}_{\mathrm{S}}} (c_{\mathrm{F}1t} - c_{\mathrm{F}2t}) \right] \frac{1 - \mathrm{e}^{-S}}{1 - E_{\mathrm{S}} \mathrm{e}^{-S}} \right\}$$
(46)

Cocurrent arrangement of the flow of phases through HFC:

 $D_{\rm F} = f(c_{\rm S0t})$ (usually a polynomial form of the rela- $D_{\rm R} = f(c_{{\rm R}0t})$ (usually a polynomial form of the relation) (43)

$$-\frac{\mathrm{d}c_{\mathrm{F}1t}}{\mathrm{d}t} = \frac{1 - \mathrm{e}^{-O}}{\tau_{\mathrm{F}} (1 + E_{\mathrm{F}})} \left(c_{\mathrm{F}1t} - \frac{c_{\mathrm{S}0t}}{D_{\mathrm{F}}} \right) \tag{17}$$

$$c_{\rm F2} = c_{\rm F1} \frac{E_{\rm F} + e^{-O}}{1 + E_{\rm F}} + c_{\rm S0} \frac{1 - e^{-O}}{D_{\rm F} (1 + E_{\rm F})}$$
(12)

$$-\frac{\mathrm{d}c_{S0t}}{\mathrm{d}t} = \tau_{S}^{-1} \left\{ c_{S0t} \frac{1 - \mathrm{e}^{-U}}{1 + E_{S}} - \frac{\dot{V}_{F}}{\dot{V}_{S}} \left(c_{F1t} - c_{F2t} \right) \times \frac{E_{S} + \mathrm{e}^{-U}}{1 + E_{S}} - c_{R0t} \frac{1 - \mathrm{e}^{-U}}{D_{R} \left(1 + E_{S} \right)} \right\}$$
(41)

$$c_{\rm S2} = c_{\rm S1} \frac{E_{\rm S} + e^{-U}}{1 + E_{\rm S}} + c_{\rm R0} \frac{1 - e^{-U}}{D_{\rm R} (1 + E_{\rm S})}$$
 (37)

$$c_{S1t} = c_{S0t} + \frac{\dot{V}_F}{\dot{V}_S} (c_{F1} - c_{F2})$$
 (32)

$$-\frac{\mathrm{d}c_{\mathrm{R}0t}}{\mathrm{d}t} = (\tau_{\mathrm{R}})^{-1} \frac{\dot{V}_{\mathrm{S}}}{\dot{V}_{\mathrm{R}}} \left\{ c_{\mathrm{R}0t} \frac{1 - \mathrm{e}^{-U}}{D_{\mathrm{R}} (1 + E_{\mathrm{S}})} - \left[c_{\mathrm{S}0t} + \frac{\dot{V}_{\mathrm{F}}}{\dot{V}_{\mathrm{S}}} (c_{\mathrm{F}1t} - c_{\mathrm{F}2t}) \right] \frac{1 - \mathrm{e}^{-U}}{1 + E_{\mathrm{S}}} \right\}$$
(48)

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SYMBOLS

interfacial area

 m^2 $\rm kmol~m^{-3}$ molar concentration initial concentration of the extracted com $c_{\rm F0}$ ponent in the feed Cdimensionless concentration $c_{\rm J}/c_{\rm F0}$ where J = F, either S, or R diffusion coefficient $D_{\rm F}$ distribution coefficient in the feed—solvent system $D_{\rm R}$ distribution coefficient in the solvent stripping solution system ddiameter of fibre $_{\mathrm{m}}$ $E_{\rm F}$ quantity defined by eqn (8) $E_{\rm S}$ quantity defined by eqn (35) \bar{E}_{F} quantity defined by eqn (23)Koverall mass transfer coefficient m s \mathbf{m}

Llength of fibres in the module Nnumber of fibres in the module $N_{\rm E}$ number of transfer units for the extraction process $N_{\rm S}$ number of transfer units for the stripping

process Oquantity defined by eqn (13)Squantity defined by eqn (34)

Uquantity defined by eqn (38)

 m^3 volume of the reservoir \dot{V} $\mathrm{m}^3~\mathrm{s}^{-1}$ flow of the liquid Wquantity defined by eqn (10)

length coordinate z

 τ space-time Θ dimensionless time

Indices

extraction F feed phase

ECextraction contactor

SCstripping contactor stripping phase

R stripping S

 $_{\rm m}$

 \mathbf{S}

- S solvent (organic phase)
- t time-dependent
- 0 marking of the place
- 1 marking of the place
- 2 marking of the place

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