# Scheduling of Multi-Product Batch Processes with Regard to Market Demands*

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A number of different products are produced on a multi-product batch line during a production campaign. Considering the limited capacity of the line, each product is to be produced in successive batches unless the amount of products demanded by customers is produced. Products are produced in a sequence that is determined by a process schedule. Expenses for cleaning of both production and intermediate storage units at a changeover to another product make the production more expensive. There are two kinds of cleaning: routine cleaning at a changeover to another batch of the same product and thorough cleaning at a changeover to another product. The algorithm searching for the optimum schedule considering market demands, the basic ideas of which are described in the contribution, enables to prove an interesting fact. The schedule with the minimum number of changeovers may paradoxically give a campaign with a greater makespan. The overall makespan for the campaign with the minimum number of changeovers is not necessarily the optimum one; surprisingly it may be greater than that for the campaign with the maximum number of changeovers.

Chemical, pharmaceutical, and food industry processes have been practised by batch ways of working for a very long time. This traditional method of work has a wide application area in the modern industry, too. Batch apparatuses are designed in standardized type series so that sufficiently flexible connection is enabled between them. Batch plants are able to respond to frequently varying market demands, important changes of technology, changing and rotating of product, and introduction of new technologies and new products. A great attention has been paid to analysis, design, scheduling, and short-term planning of batch processes. High prices of final products of batch processes provoked efforts at improving of employment of batch production lines. It turns out that very simple organizational steps can decrease the completion time of a batch process and consequently increase its productivity.

Batch processing tasks can be divided in a few types according to the process structure (see Fig. 1) and way of process organization [1]: multi-product plant design; multi-purpose plant capacity planning; short-term planning and scheduling.

It is an important fact that analysis of batch processes does not go into detail of elements of batch processes. They are supposed to be sufficiently described with a few integral parameters (processing times, cleaning times, batch sizes). On the other hand, interactions between elements of a batch process (apparatuses, operations running on them, storage units) are very important. The optimum behaviour of the whole process is reached by the optimum synchronizing of the interactions by means of both discrete events modelling and scheduling. The parameters of the process elements are considered steady from the point of view of scheduling and set up to good suboptimum values in advance. These values are stated in the instruction sheet of the batch process for deterministic processes. For stochastic processes, they are usually averaged from measured process data.

In plants of this type, several products are produced in sequential campaigns. They are produced in the order that is determined in advance. Operations run on firmly assigned apparatuses. Design of batch plant the products of which are produced in campaigns in some sequence is implemented on one hand, and on the other hand, the optimum sequence of the campaign processes that minimizes the makespan of completing of the last product in the campaign is searched. Most of methods are aimed at entire employment of apparatuses and removing or minimization of their stoppage at least.

A typical feature of multi-purpose batch plants

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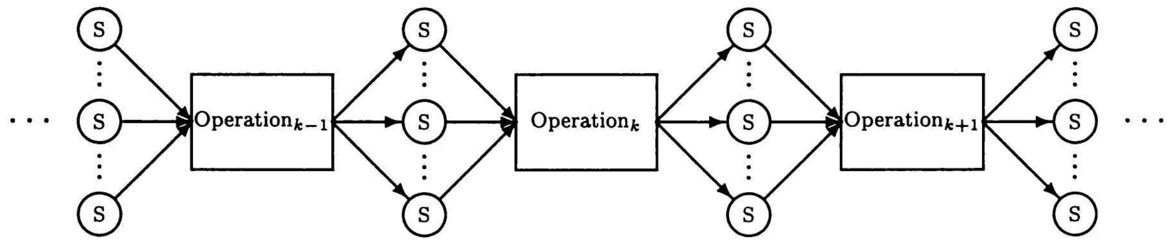


**Fig. 1.** Technological chart of a linear multi-product batch process. S – intermediate storage units.

processes is simultaneous production of more products. A set of apparatuses and equipment items is available in the plant. Some of them are special; others are universal. The product routes through them have to be determined. Several products are produced simultaneously and the same product may follow different routes through the plant equipment at different times. The basic criterion of the production strategy design is the maximum employment of all the apparatuses.

Needs for modification or improvement of batch production lines are more frequent than those for design of new plants. The basic idea of short-term planning and scheduling are mutual exchanges and permutations of product order, campaigns, operations, apparatuses, and equipment items. The goal is to get the sequence or choice of these elements so that the process may become optimal, usually from the time point of view. A number of other conditions may have to be fulfilled (minimization of storage costs, minimization of the product makespan, increasing of equipment employment, good manufacturing practice, energy and material limits, *etc.*).

## THEORETICAL

### Models of Multi-Product Batch Process

The presented model of multi-product batch processes is based on the following assumptions [2]:

– All products are produced in the same order on each processing unit.

– Processing of a product on an apparatus may not be interrupted and then resume later on.

– An apparatus may not process more than one product at a time, nor a product may be processed by more than one unit simultaneously.

– Storage items are always available after the last processing units.

– Intermediate storage units of four different types may be available between apparatuses:

UIS: unlimited intermediate storage
FIS: finite intermediate storage
NIS: no intermediate storage
ZW: zero waiting-time intermediate storage

– The intermediate storage is measured in terms of the number of units and not the physical size of storage (each storage unit can temporarily hold any product batch).

– A product batch may be temporarily held in the processing unit if the unit is linked up with the UIS, FIS or NIS intermediate storage. If linked intermediate storage is of the ZW type, the product batch has to be transferred immediately to the next processing units.

– The transferring time of batches between apparatuses and storage units is included into the processing time. In addition to that, the starting time of processing unit and the time needful for routine cleaning of the apparatuses and storage units between two equal products is usually taken in the processing time as well.

Based on these ideas, *Ku* and *Karimi* [3] formulated a mathematical model for the linear – serial batch processes. They derived the recurrence relations for computing the times at which products leave processing units. Topology of the great majority of batch processes is more complicated than Ku and Karimi have considered. Usual topology of processes is not linear only but it may also contain the following substructures (Fig. 2):

– *bifurcation* or *ramification* – parts of processed mass are processed in different ways (processing of side products or waste stuffs, regeneration of auxiliary stuffs),

– *connection* – a few different raw materials are processed,

– *parallel branches* – processed material is distributed among several equal apparatuses for capacity reasons.

The terms *previous* and *following* are defined unambiguously in all respects for linear – serial batch processes. For describing of branching batch processes, a directed graph has to be used. Then, a set $H$ of edges of this directed graph describes mutual linkage of the operations [4]. Each edge starts in a vertex corresponding to an operation and ends in a vertex that corresponds to some of subsequent operations. It is evident that the completion time of the $i$-th product processing on the $j$-th apparatus is not necessarily equal to the time $T_{i,j}$ at which the $i$-th product leaves the $j$-th apparatus. Processing of a product may be started only if the following assumptions are satisfied:

– All previous operations have been finished by now.

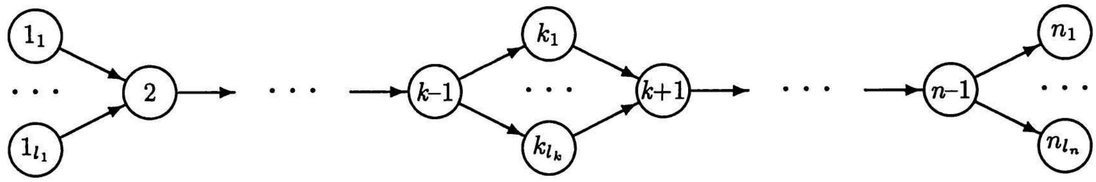


**Fig. 2.** Scheme of a branching batch process (connection of branches, parallel branches, ramification of branches).

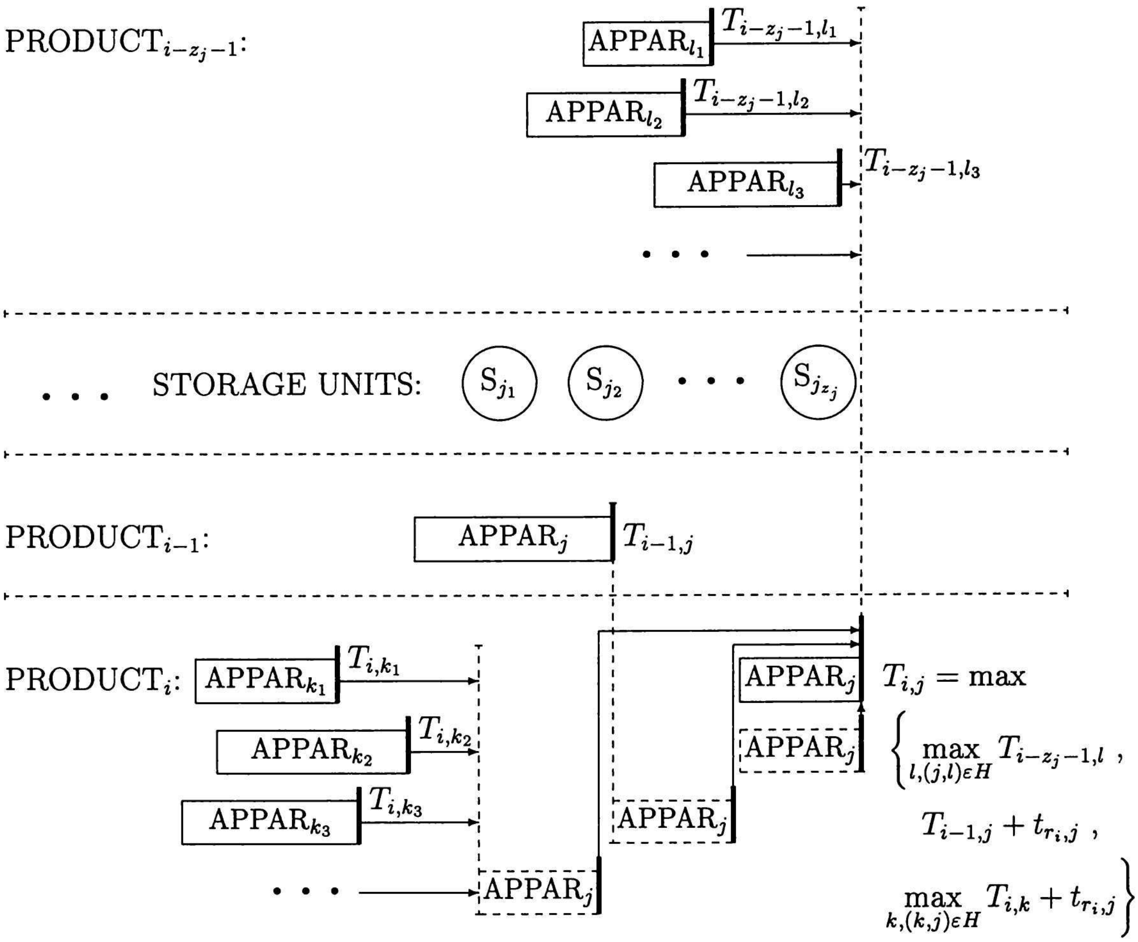


**Fig. 3.** Scheme of determination of the time $T_{i,j}$ at which $i$-th product leaves $j$-th apparatus.

– The same operation processing the previous product has been finished on the same unit by now.

– The processed previous product has been transferred into the following processing or free storage units. If the following unit is occupied by some of the previous products, the product is temporarily kept in the unit on which it has just been processed.

Then recurrence relations (Fig. 3) for computing of the completion times $T_{i,j}$ at which the $i$-th product leaves the $j$-th apparatus and the makespan $T_{M,N}$ of a multi-product process ($r_i$ is a serial number of $i$-th product according to the schedule, $t_{r_i,j}$ is its processing time on $j$-th apparatus, $z_j$ is the number of intermediate storage units after $j$-th apparatus, $M$ is the number of products, $N$ is the number of operations) can be formulated in this way

$$T_{i,j} = \max \left\{ \max_{k,(k,j)\in H} T_{ik}, T_{i-1,j}, \max_{k,(k,j)\in H} T_{i-z_j-1,k} - t_{r_i,j} \right\} + t_{r_i,j} \qquad (1)$$

for $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, N$, or $T_{i,j} = 0$ if values of $i$ or $j$ are out of range.

## Short-Term Planning of Batch Campaigns with Regard to Market Demands

A number of different products are produced on a multi-product batch line during a production campaign. Considering the limited capacity of the line, each product is to be produced in successive batches unless the amount of products demanded by market is produced. Products are produced in a sequence that is determined by a schedule of the process. The optimum schedule, which fulfils requirements for amount and assortment of products and as far as possible the required delivery schedule, can be computed. Customer requirements for successive supplies, limited packing, and storage capacity cause that it is impossible to produce the same product in the only continual series of batches. It must be produced in a few small series that do not follow up with each other. Expenses for cleaning of both production and intermediate storage units at a changeover to another product make the production more expensive. There are two kinds of cleaning:

– Routine cleaning at a changeover to another batch of the same product – it can be included into the processing time of a production operation itself because it is carried out after each operation and its length depends on the only type of product. A simple rinse of the units is sometimes adequate; sometimes the unit must not be cleaned at all.

– Thorough cleaning at a changeover to another product that is more complex, time-demanding and above all it depends on two products, the first of them has just been finished and the second is waiting to be started. A different cleaning operation that has determined its own cleaning period must be carried out for each type of changeovers.

## Enlargement of the Batch Process Model Regarding Market Demands

The products are produced in the course of a production campaign according to a schedule by batches in a sequence of subcampaigns. A case when the same product is not produced en bloc but in several subcampaigns that do not follow up with each other is frequent. It is necessary to complete the above model of the multi-product batch process with the following assumptions to respond to market demands:

– The batch size is given for each product by means of the mass amount of final product that can be produced by an only batch. This parameter is usually appended to the product processing times.

– Cleaning of a production line in the course of changeover between two different products is given as a sequence of special operations – their "product" is a clean line ready to start the production of a product that is subsequent according to the schedule. The number of cleaning operations is equal to the number of possible changeovers between products $M(M-1)$.

– The changeovers between subcampaigns that produce the same product must be penalized in order to eliminate their combination. Otherwise, a pair of subcampaigns producing the same product that must not be carried out in sequence could not be put together by the scheduling algorithm. A simple way of solution of the problem is introduction of another strongly handicapped operation that should have to be carried out in the case if two subcampaigns of the same type followed one after another.

## A Campaign Creation

Customer requirements for the scheduling algorithm needs are given in a simplified form. A customer order has the following form: the sort of the product; the required amount of the product; the delivery date of required product.

The required number of batches for each subcampaign is calculated by means of the known batch size of each product; the results are rounded up. Then the subcampaigns are ordered according to delivery dates. If several subcampaigns of the same product are created by that, they are put together to create another greater one. It is proper to consider rounding during the calculations of the numbers of batches and as the case may be, the batch number of the subcampaign is reduced. The scheduling algorithm searches for the optimum order of subcampaigns. In the course of calculations of the campaign makespan for the determined subcampaign schedule, the sequence of batches of products and changeovers between successive products is generated to calculate the campaign makespan. The dates of product delivery can be computed from the completion times of the subcampaigns. If these dates differ from the required ones, both alternative arrangement of them and synchronization of customer requirements with the ability of the production line to produce them must be done.

# EXPERIMENTAL

A simple model batch process consists of 6 production units, on which may run 6 operations. Operations have already firmly assigned apparatuses. The basic parameters of the process are shown in Table 1. Processing times for all products and apparatuses and batch sizes are shown in Table 2. Cleaning times of production and storage units for all possible changeovers are shown in Table 3.

Nonpermissible changeover between the same products in different successive subcampaigns is penalized

**Table 1.** Basic Parameters of the Model Multi-Product Batch Process

| | |
|---|---|
| Number of operations $N$ | 6 |
| Number of products $M$ | 3 |
| Number of storage units $z_j$ | 1 — 1 — 1 — 1 |

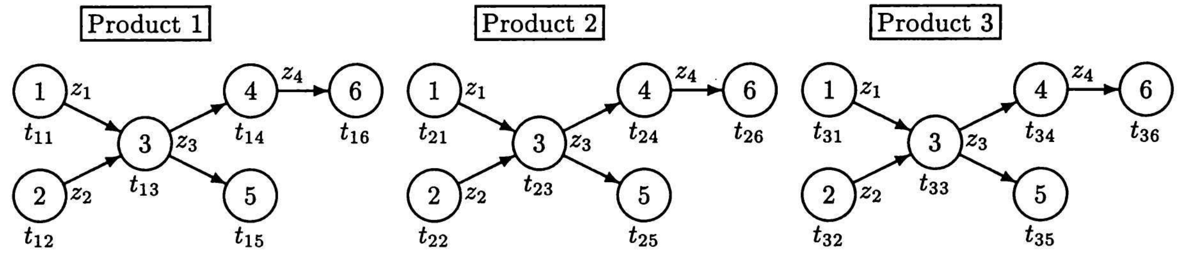


**Fig. 4.** Scheme of the model multi-product batch process ($t_{ij}$ is the processing time of $i$-th product on $j$-th apparatus, $z_j$ is the number of storage units after $j$-th apparatus).

**Table 2.** Processing Times and Batch Sizes of the Model Multi-Product Batch Process

| Product | Processing times of products on units $t_{ij}$/h | | | | | | Batch size |
|---|---|---|---|---|---|---|---|
| | $U_1$ | $U_2$ | $U_3$ | $U_4$ | $U_5$ | $U_6$ | $q_i$/kg |
| $P_1$ | 12 | 22 | 6 | 30 | 33 | 15 | 60 |
| $P_2$ | 14 | 21 | 15 | 30 | 5 | 10 | 80 |
| $P_3$ | 15 | 15 | 26 | 13 | 9 | 25 | 75 |

**Table 3.** Cleaning Times of the Model Multi-Product Batch Process

| Changeover | Cleaning times of production units $t'_{ij}$/h | | | | | |
|---|---|---|---|---|---|---|
| | $U_1$ | $U_2$ | $U_3$ | $U_4$ | $U_5$ | $U_6$ |
| $P_1 \to P_2$ | 2 | 6 | 3 | 3 | 5 | 8 |
| $P_1 \to P_3$ | 6 | 5 | 5 | 2 | 1 | 2 |
| $P_2 \to P_1$ | 2 | 4 | 3 | 2 | 1 | 3 |
| $P_2 \to P_3$ | 4 | 5 | 3 | 6 | 4 | 2 |
| $P_3 \to P_1$ | 5 | 4 | 2 | 4 | 3 | 3 |
| $P_3 \to P_2$ | 3 | 2 | 8 | 2 | 2 | 3 |

for each operation by sum total of cleaning times for all product changeovers. Mutual connection of operations is the same for all products and together with both the processing time and storage number assigned to the operations is shown in Fig. 4.

The given demands of customers and corresponding partial productive campaigns are summarized in Table 4. It is planned to produce 7 required batches in 5 successive subcampaigns altogether.

**Table 4.** Summary of Short-Term Market Demands for Produced Products

| Subcampaign | Product | Demanded amount kg | Number of batches |
|---|---|---|---|
| A | $P_1$ | 120 | 2 |
| B | $P_2$ | 80 | 1 |
| C | $P_3$ | 150 | 2 |
| D | $P_1$ | 60 | 1 |
| E | $P_2$ | 80 | 1 |

**Table 5.** Batch and Subcampaign Makespans

| Subcampaign | Batch | Product | Batch makespan h | Subcampaign makespan h |
|---|---|---|---|---|
| A | 1 | $P_1$ | 73 | |
| | 2 | $P_1$ | 103 | 103 |
| B | 1 | $P_2$ | 134 | 134 |
| C | 1 | $P_3$ | 168 | |
| | 2 | $P_3$ | 193 | 193 |
| D | 1 | $P_1$ | 213 | 213 |
| E | 1 | $P_2$ | 241 | 241 |

## RESULTS AND DISCUSSION

The optimum schedule calculated with the help of the genetic algorithm [5] is figured by a well-arranged way by means of a Gantt chart in Fig. 5, the batch completion times and the campaign makespans are shown in Table 5.

The delivery dates can be determined for the above data. In the case of their variance with required ones, new negotiations with customers may be carried out to modify requirements. It is possible to prefer some order over the other ones by a choice of penalties so that it is always chosen as the first one by the scheduling algorithm. Alternatively, some other order can be penalized in such a way that it is always chosen as the last one in the campaign. The optimum schedules usually are not unique and it is possible to choose from them the schedule that best meets the customer requirements. Four optimum schedules that are equivalent from the campaign makespan point of view are found in this example. They result by mutual exchanges of subcampaigns A and D, or B and E, or the both exchanges alternatively.

The algorithm searching for the optimum schedule considering market demands the basic ideas of which

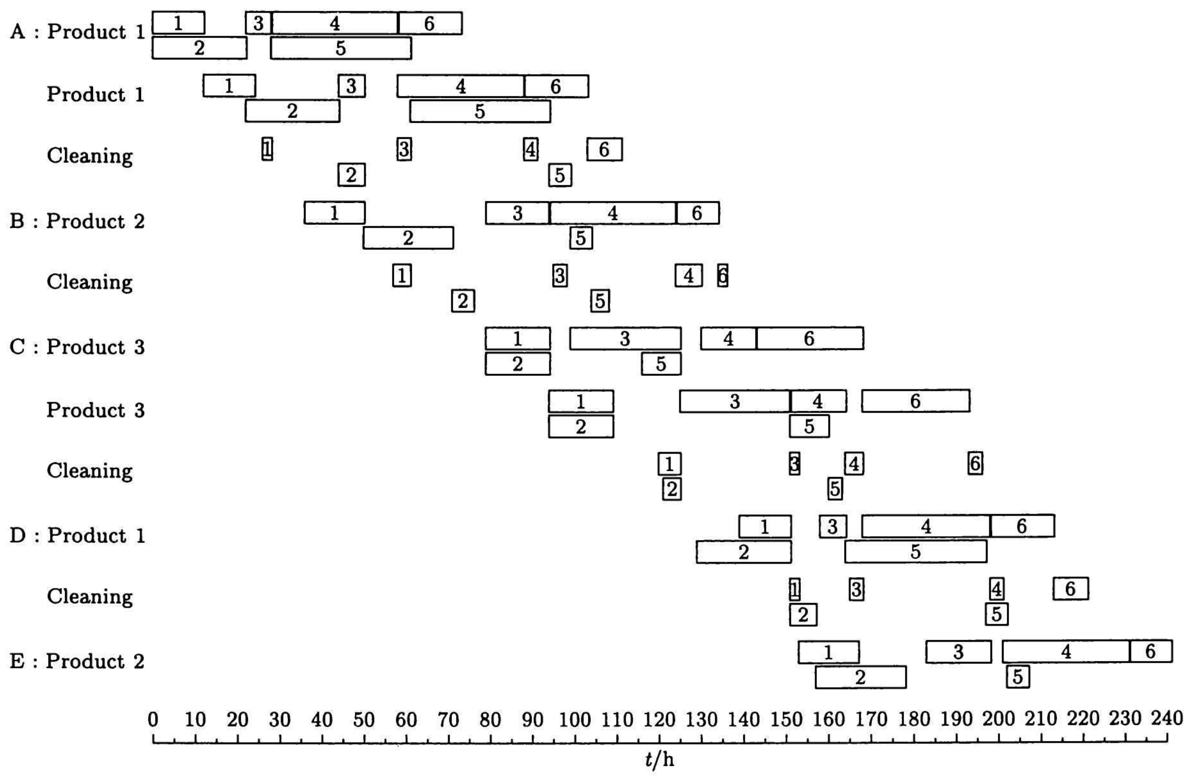


**Fig. 5.** Gantt chart of the model multi-product batch process satisfying the short-term market demands.

**Table 6.** Possibilities of Campaign Organization Considering Demanded Amount of Products

| Campaign | Number of subcampaigns | Maximum number of batches in subcampaigns | | | Campaign makespan |
|---|---|---|---|---|---|
| | | $P_1$ | $P_2$ | $P_3$ | h |
| (22)—(33)—(111) | 3 | 3 | 2 | 2 | 245 |
| (3)—(111)—(3)—(22) | 4 | 3 | 2 | 1 | 246 |
| (111)—(2)—(33)—(2) | 4 | 3 | 1 | 2 | 245 |
| (111)—(3)—(2)—(3)—(2) | 5 | 3 | 1 | 1 | 245 |
| (11)—(33)—(1)—(22) | 4 | 2 | 2 | 2 | 240 |
| (11)—(3)—(1)—(3)—(22) | 5 | 2 | 2 | 1 | 234 |
| (11)—(2)—(33)—(1)—(2) | 5 | 2 | 1 | 2 | 241 |
| (11)—(3)—(1)—(2)—(3)—(2) | 6 | 2 | 1 | 1 | 239 |
| (1)—(22)—(1)—(33)—(1) | 5 | 1 | 2 | 2 | 245 |
| (1)—(3)—(1)—(3)—(1)—(22) | 6 | 1 | 2 | 1 | 237 |
| (1)—(2)—(1)—(33)—(1)—(2) | 6 | 1 | 1 | 2 | 243 |
| (1)—(2)—(1)—(3)—(1)—(3)—(2) | 7 | 1 | 1 | 1 | 238 |

are mentioned above enables to prove an interesting fact. The routine approach to the optimum schedule creation that usually minimizes changeover times with the help of searching for the optimum Hamiltonian path in a changeover graph need not lead to the goal. In the above example, it can be shown that the schedule with the minimum number of changeovers may paradoxically give a campaign with a greater makespan. The optimization of all possible campaigns has been carried out starting with production of all batches of each product in the only campaign (the minimum duration of changeovers and cleaning operations as well) and ending with production of each batch in other subcampaign (the maximum duration of changeovers). The overall makespan for the campaign with the minimum number of changeovers is not the optimum one, but surprisingly it is greater than that for the campaign with the maximum number of changeovers (Tables 6 and 7).

Almost triple duration of cleaning operations is spread over the whole process in such a way that some unneeded pauses and waits for finishing of an operation or releasing of a unit are fulfilled. The process as a whole lasts shorter time although the sum of pro-

**Table 7.** Comparison of Possible Campaigns Considering Makespan and Cleaning Time Sum

| Campaign | Campaign makespan h | Time savings % | Number of cleanings | Sum total of cleaning times h | Relative value of cleaning time sum |
|---|---|---|---|---|---|
| (22)—(33)—(111) | 245 | 0.00 | 2 | 45 | 1.00 |
| (3)—(111)—(3)—(22) | 246 | −0.41 | 3 | 66 | 1.47 |
| (111)—(2)—(33)—(2) | 245 | 0.00 | 3 | 75 | 1.67 |
| (111)—(3)—(2)—(3)—(2) | 245 | 0.00 | 4 | 93 | 2.07 |
| (11)—(33)—(1)—(22) | 240 | 2.04 | 3 | 69 | 1.53 |
| (11)—(3)—(1)—(3)—(22) | 234 | 4.49 | 4 | 87 | 1.93 |
| (11)—(2)—(33)—(1)—(2) | 241 | 1.63 | 4 | 99 | 2.20 |
| (11)—(3)—(1)—(2)—(3)—(2) | 239 | 2.45 | 5 | 117 | 2.60 |
| (1)—(22)—(1)—(33)—(1) | 245 | 0.00 | 4 | 84 | 1.87 |
| (1)—(3)—(1)—(3)—(1)—(22) | 237 | 3.27 | 5 | 111 | 2.47 |
| (1)—(2)—(1)—(33)—(1)—(2) | 243 | 0.82 | 5 | 111 | 2.47 |
| (1)—(2)—(1)—(3)—(1)—(3)—(2) | 238 | 2.86 | 6 | 129 | 2.87 |

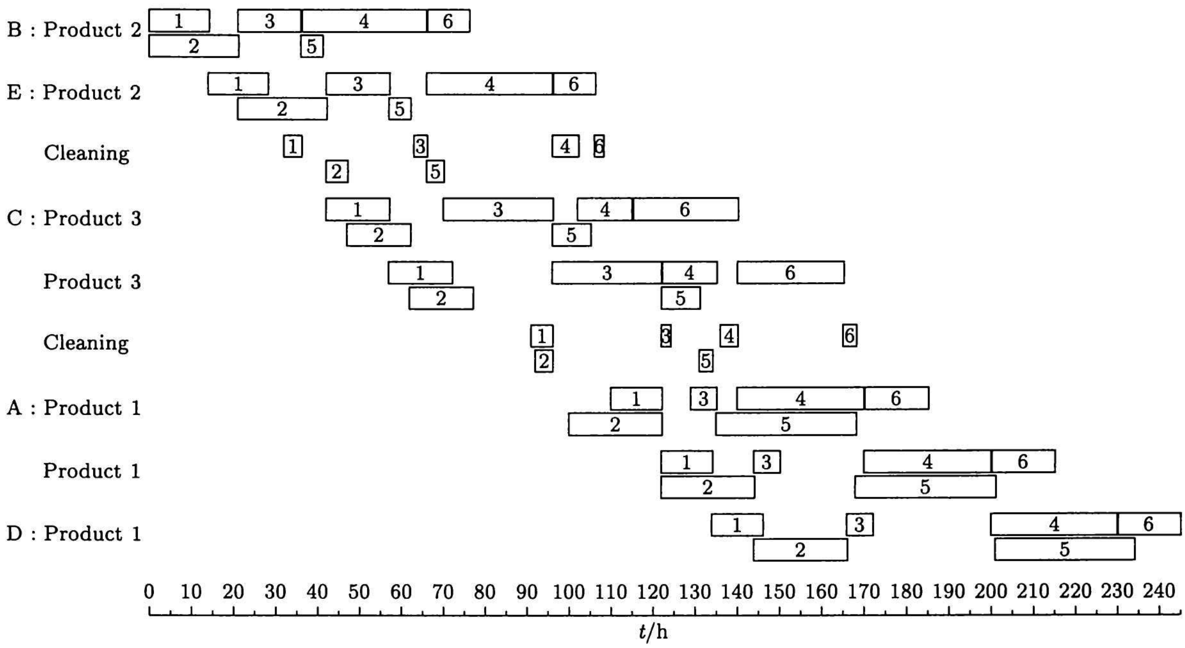


**Fig. 6.** Gantt chart of the model multi-product batch process minimizing the changeovers.

cessing and cleaning times is greater by 84 h. It is necessary to decide in a particular case which criterion is essential:

– the reduction of the campaign makespan which improves the productivity of batch line,

– the cleaning span and number of cleaning operations that influences manpower (insignificant regarding the fact that present manpower is sufficient because it is better employed only) and auxiliary means and powers for cleaning (important).

The Gantt chart of the campaign that minimizes changeovers is shown for comparison – all batches of each product are produced by the only subcampaign (22)—(33)—(111) (Fig. 6). The Gantt chart of the optimum campaign (11)—(3)—(1)—(3)—(22) is shown in Fig. 7. This campaign minimizes the makespan and improves exploitation of production line and its productivity. The number of changeovers is double in comparison with the campaign with the minimum number of changeovers and the duration of cleaning is nearly twice longer.

## CONCLUSION

The main problems of creation of short-term plans and schedules for branching multi-product batch processes that should satisfy short-term market demands are connected with their grouping into a sequence of corresponding product campaigns. Requirements of customers on successive supplies of products in requisite dates and amounts force producers to make a short-term plan composed of shorter productive cam-

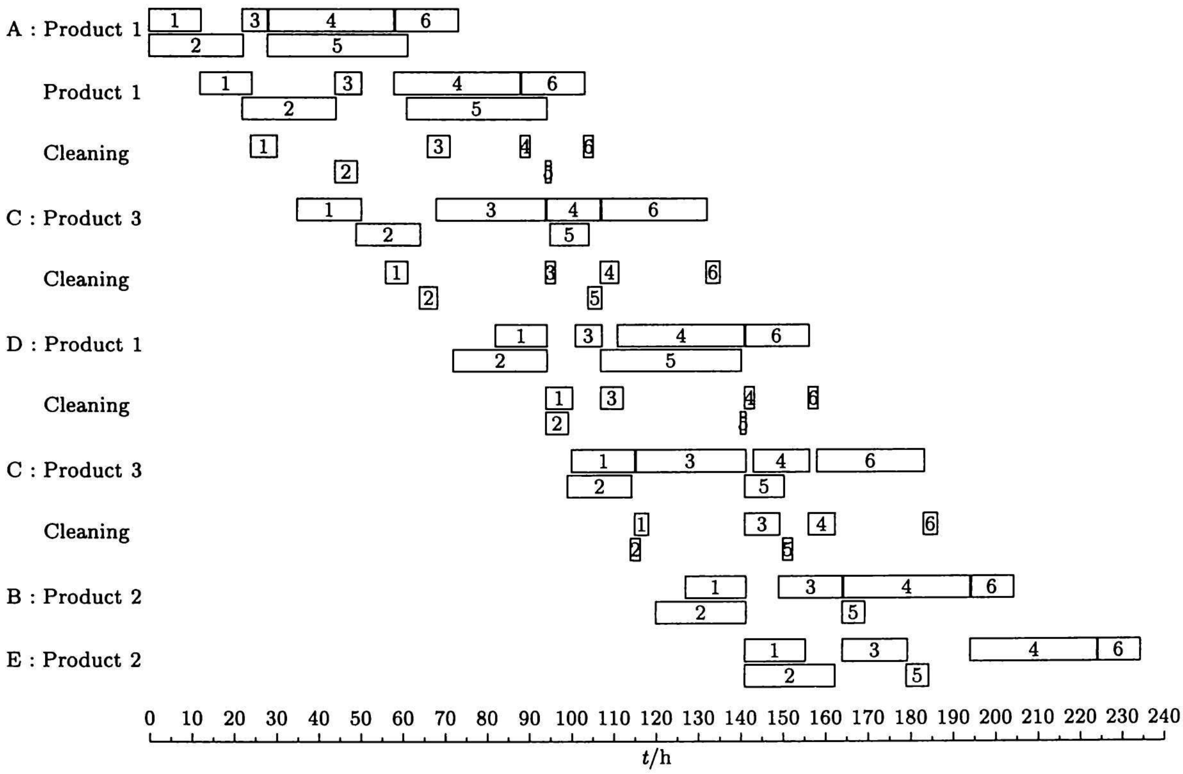


**Fig. 7.** Gantt chart of the optimum model multi-product batch process.

paigns. Considering these demands it is not possible to create long-term optimum campaigns.

In view of taking into account frequent changes of products, processes of cleaning of apparatuses before a changeover to another product are of considerable importance. A short-term production plan that minimizes the number of changeovers is not necessarily the optimum one. Branching multi-product batch processes have an extensive number of freedom degrees in location of operations in time. This fact enables to insert cleaning operations between the productive ones in such a way that the makespan increases less than it is expected. The optimum schedule of a multi-product batch process, which is computed by efficient combinatorial optimization, usually contains more changeovers between products than it would be expected after a cursory analysis.

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