# Filtration of Signals

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The contribution deals with the digital filter design for the filtration of concentration sensors output signals. The data from the $CO_2$ analyzer were filtered with FIR and IIR digital filters. The FIR filter is also called a nonrecursive or a convolution filter. Its transfer function is a polynomial in $z^{-1}$, it has no poles. The IIR filter is also called a recursive one and has a rational transfer function in $z^{-1}$, it has both poles and zeros. A very important set of methods for IIR filter design is based on converting Butterworth, Chebyshev, and elliptic-function analogue filters to digital ones. The best results in the filtration of $CO_2$ concentration were obtained with elliptic and FIR filters.

The economic aspect plays a great role in the judgement of production processes nowadays. The preparation of the gas mixture with the requested concentration enables us to rationalize production processes and decrease the financial and the energetic pretentiousness of the production [1].

Keeping the requested concentration of the mixture is not a simple task. The requirements for the special precision and stability of the prepared mixture appear in the laboratory practice often, too. An apparatus for the preparation of gas mixtures according to [2] was created in our department. The precision depends on the stabilization of input and output pressures and on the stabilization of the frequency of the dosing during the preparation of the mixture on this apparatus. In the case when the stabilization of the output pressure is not available or it has bad quality, it is possible to prepare the gas mixture by controlling the requested concentration measured by the exact automatic analyzer. The signal from the automatic analyzer is processed by A/D converter while controlling the system by the computer. Various sensors, converters, and other devices are the source of noises and errors. The results of measurements and their processing need the signal filtration. Using the filter means the simplification of the control algorithm and greater precision of the prepared concentration.

## EXPERIMENTAL

Fig. 1 represents a real control scheme for the measurement and control of the gaseous mixture concentration.

This circuit is for the measurement and control of $CO_2$ concentration in the air. The air gets into the system from the compressor (the local laboratory transport). The pressure bomb is the source of $CO_2$. The food $CO_2$ is used, it is distributed in high-pressure bottles. The pressure of gases incoming to the container is stabilized according to the regular activity of the whole system. The gases are delivered to the mixing container, where the homogenization of the gas mixture starts. The analyzer based on the measurement of the diffusivity of the gas mixture is used to obtain the concentration of $CO_2$. The system is controlled by the computer and it is necessary to identify it. The simple method to estimate the parameters is identified by the measured response characteristic.

We obtain a transfer function from the measured transient response. This system can be described as the first-order system with time delay.

The transfer function is given by the equation

$$H(s) = \frac{K}{Ts+1}e^{-\tau_d s} \qquad (1)$$

with a time delay $\tau_d = 10$ s, gain $K = 1$, and time constant $T = 5$ s.

The output signal is corrupted by noise. The measured data of the $CO_2$ concentration were filtered with FIR and IIR digital filters. This problem was solved in MATLAB/SIMULINK environment [3].

## RESULTS AND DISCUSSION

### FIR Filters

Digital filters with a finite duration impulse response can reach an exact linear phase and cannot be unstable [4]. The transfer function of a filter can be written as a polynomial, it has no poles. Because of the method of implementation, the FIR filter is also called a nonrecursive filter or a convolution filter. From the time domain view of this operation, the FIR filter is sometimes called a moving average filter.

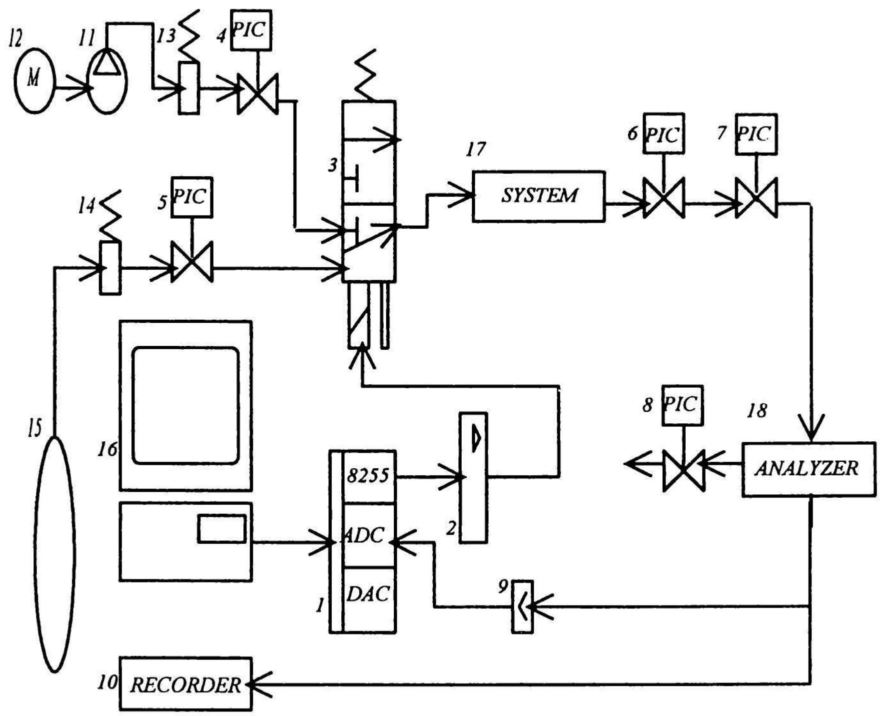


**Fig. 1.** The controlled system scheme: *1.* Transmitter SHS-A, *2.* amplifier, *3.* bistable valve (Festo), *4.* air input pressure stabilizer, *5.* $CO_2$ input pressure stabilizer, *6.* mixture pressure stabilization and measurement, *7.* rotameter, *8.* output pressure stabilizer, *9.* output signal amplifier, *10.* signal recorder, *11.* compressor, *12.* compressor motor, *13.* air reduction station, *14.* $CO_2$ reduction station, *15.* pressure bottle with $CO_2$, *16.* PC AT-286/1M/40HD, *17.* container, *18.* $CO_2$ analyzer.

**Table 1.** Competent Transfer Functions of FIR Filters

| Type of filter | 1st order transfer function | 4th order transfer function |
|---|---|---|
| FIR | $z + 1$ | $z^4 + 8.343z^3 + 16.52z^2 + 8.343z + 1$ |

The output of these filters can be written as a finite convolution sum

$$y(n) = \sum_{i=0}^{N} b_i x(n-i) \qquad (2)$$

The transfer function of a FIR filter is given by

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_N z^{-N} = \sum_{i=0}^{N} b_i z^{-i} \qquad (3)$$

Fig. 2 shows the time dependence for the measured unfiltered signals and signals with the first- and fourth-order low pass FIR filter with cut off frequency $\omega_n = 1/T = 0.2\ \text{s}^{-1}$ and sampling period $T_s = 1$ s.

The competent transfer functions of FIR filters are in Table 1.

## IIR Filters

The digital filter with an infinite impulse response is called a recursive filter or autoregressive moving average filter [4]. In contrast to the FIR filter with a polynomial transfer function, the IIR filter has a rational function. The transfer function being a ratio of polynomials means it has finite poles as well as zeros.

The defining relationship between the input and output variables for the IIR filter is given by

$$y(n) = \sum_{i=0}^{M} a_i x(n-i) - \sum_{i=1}^{N} b_i y(n-i) \qquad (4)$$

The transfer function is also the ratio of the $z$ transforms

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{i=0}^{M} a_i z^{-i}}{1 + \sum_{i=1}^{N} b_i z^{-i}} \qquad (5)$$

An IIR filter has an advantage over an FIR filter in that it generally has fewer coefficients than an FIR filter with similar magnitude characteristics. In applications where the coefficients of the filter are updated in real time, the advantage of fewer coefficients in the IIR filter may be significant.

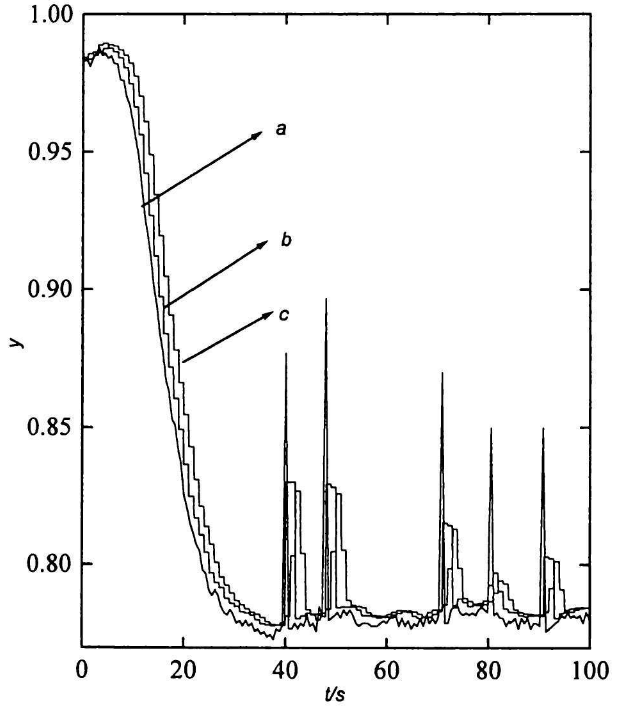


**Fig. 2.** Time dependence of the 1st and 4th order FIR filter: $y$ – mole fraction of $CO_2$, *a*) unfiltered signal, *b*) signal with the 1st order filter, *c*) signal with the 4th order filter.

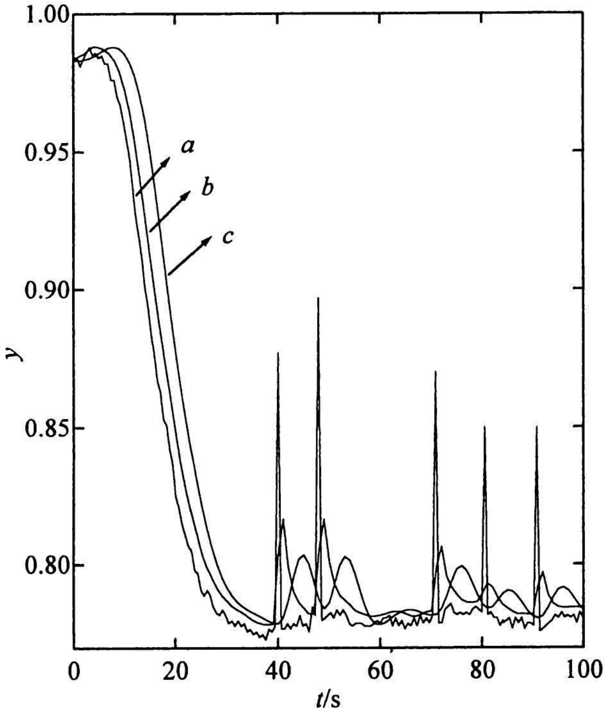


**Fig. 3.** Time dependence of the 1st and 4th order low pass Butterworth filter (the same meaning of symbols as in Fig. 2).

In our experiment, the following 1st and 4th order low pass digital IIR filters were applied: Butterworth filter, Chebyshev type I and Chebyshev type II filter, and elliptic or Cauer filter. The cut off frequency $\omega_n = 1/T = 0.2\ s^{-1}$ and sampling period $T_s = 1$ s were the same as for the FIR filter. The Chebyshev type I filter was with $R_p = 0.5$ dB ripple in the passband, the Chebyshev type II filter was with the stopband ripple $R_s = 5$ dB down, the elliptic filter had the same $R_p = 0.5$ dB and $R_s = 5$ dB.

Figs. 3—6 show the time dependence for the measured unfiltered signals and signals with the 1st and 4th order low pass digital Butterworth, Chebyshev type I, Chebyshev type II, and elliptic filters. The competent transfer functions of IIR filters are in Table 2.

It is reasonable, as can be seen from the figure as well, when the range of the filter is increasing, the filtering of the unavailable signals goes better with both FIR and IIR filters. The process dynamics slightly goes down, however. We can obtain a simple empirical filter for this concrete system by using physical maintenance. It has been detected observing the signal from the converter that the difference between 5 values should not be bigger in the absolute value than 2. In the other case there is a failure on the converter. According to this the data were filtered so that the 5 values were compared with each another, the difference was calculated and if its absolute value was bigger than 2, the outlying value was eliminated and the arithmetic average was counted from the others.

**Table 2.** Competent Transfer Functions of IIR Filters

| Type of filter | 1st order transfer function | 4th order transfer function |
|---|---|---|
| Butterworth | $\frac{0.2452z + 0.2452}{z - 0.5095}$ | $\frac{0.004824z^4 + 0.0193z^3 + 0.02895z^2 + 0.0193z + 0.004824}{z^4 - 0.37z^3 + 2.314z^2 - 1.055z + 0.1874}$ |
| Chebyshev type I | $\frac{0.4819z + 0.4819}{z - 0.03618}$ | $\frac{0.0024z^4 + 0.0099z^3 + 0.0149z^2 + 0.0099z + 0.0024}{z^4 - 2.914z^3 + 3.518z^2 - 2.035z + 0.4729}$ |
| Chebyshev type II | $\frac{0.181z + 0.181}{z - 0.6381}$ | $\frac{0.4134z^4 - 0.7789z^3 + 1.036z^2 - 0.7789z + 0.4134}{z^4 - 1.991z^3 + 1.924z^2 - 0.8975z + 0.2698}$ |
| Elliptic | $\frac{0.4819z + 0.4819}{z - 0.03618}$ | $\frac{0.4993z^4 - 1.45z^3 + 2.039z^2 - 1.45z + 0.4993}{z^4 - 2.884z^3 + 3.701z^2 - 2.323z + 0.652}$ |

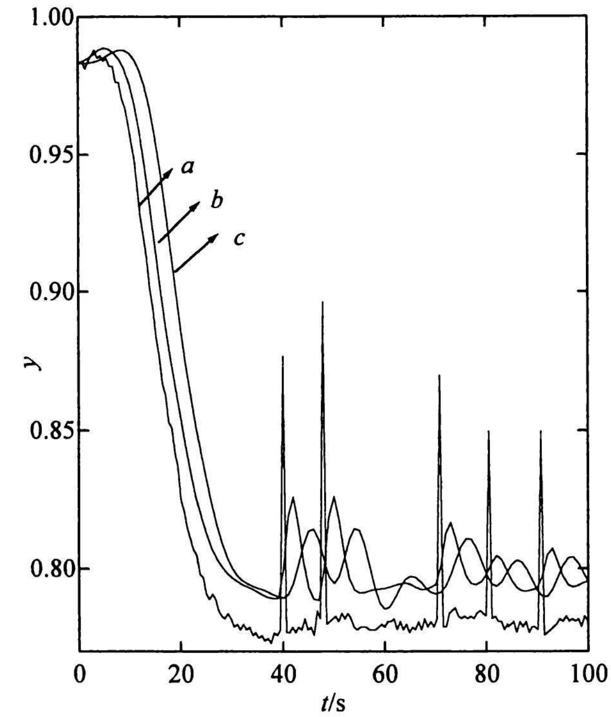


**Fig. 4.** Time dependence of the 1st and 4th order low pass Chebyshev type I filter (the same meaning of symbols as in Fig. 2).

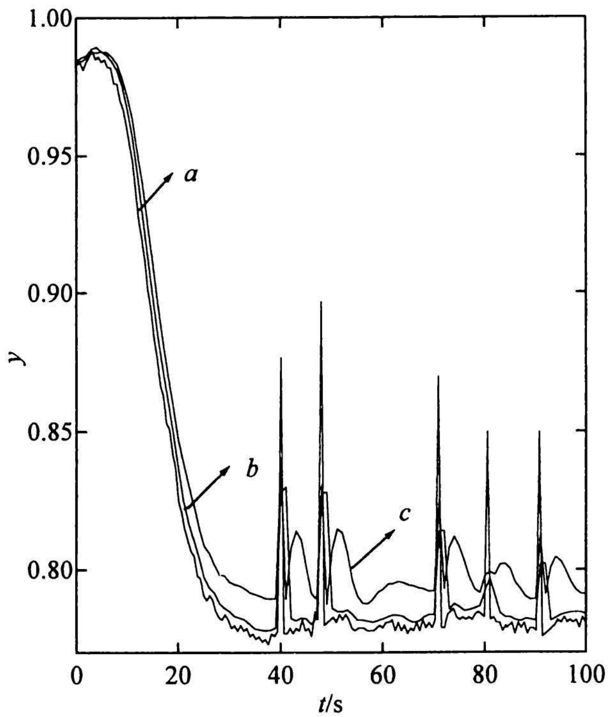


**Fig. 6.** Time dependence of the 1st and 4th order low pass elliptic filter (the same meaning of symbols as in Fig. 2).

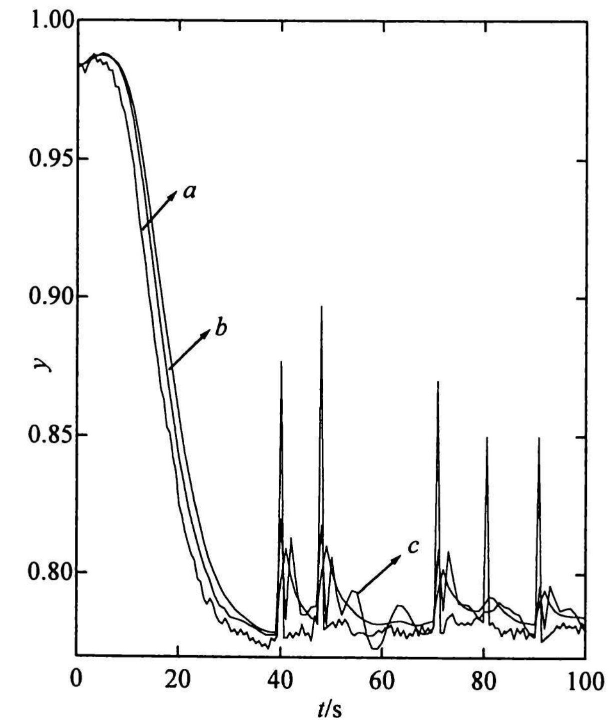


**Fig. 5.** Time dependence of the 1st and 4th order low pass Chebyshev type II filter (the same meaning of symbols as in Fig. 2).

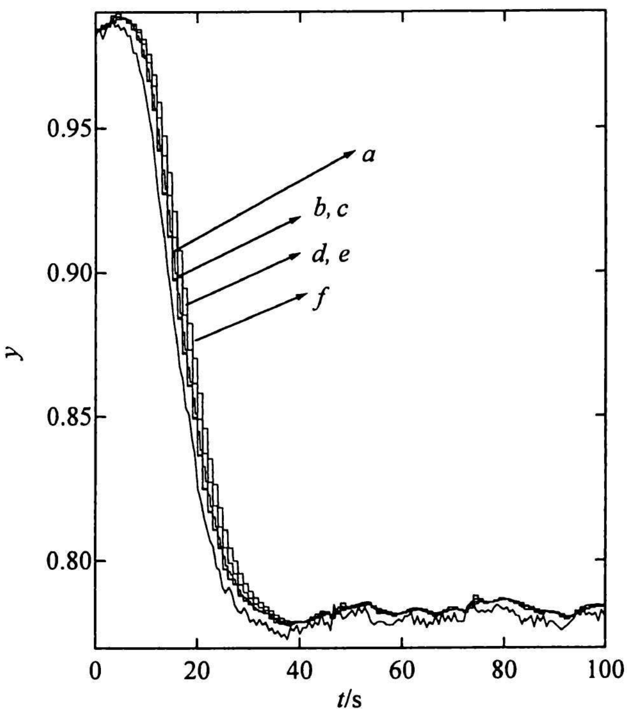


**Fig. 7.** Time dependence of the modified unfiltered signal (*a*) and signals with the 1st order elliptic (*b*), FIR (*c*), Chebyshev type I (*d*), Butterworth (*e*), and Chebyshev type II (*f*) filters.

The use of the filter of the 1st range is sufficient for such modified values. The elliptic and FIR filters seem to be the most suitable, then goes Chebyshev, Butterworth, and the inverted Chebyshev filter, as seen in

Fig. 7, where the modified behaviour of the measured signal and the signal filtered by the mentioned filters is shown. The behaviours of the elliptic and FIR filters signals merge.

## CONCLUSION

When it is necessary to filter the inputs and the outputs of the real processors, we can use digital FIR or IIR filters. It became useful in our matter to eliminate the measurements loaded by greater mistakes at first, and then use FIR, respectively IIR filter. The filter of the 1st range is sufficient.

## SYMBOLS

| | | |
|---|---|---|
| $H(s), H(z)$ | transfer functions | |
| $t$ | time | s |
| $y$ | mole fraction of $CO_2$ | |
| $K$ | gain | |
| $T$ | time constant | s |
| $\tau_d$ | time delay | s |
| $T_s$ | sampling period | s |
| $\omega_n$ | cut off frequency | $s^{-1}$ |
| $R_p$ | passband ripple | dB |
| $R_s$ | stopband ripple | dB |

## REFERENCES


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