Dynamic modelling of activated sludge process
III. Statistical model

[a]M. KRÁLIK, [b]J. DERCO, [b]J. FARKAŠ, and [b]P. GUBOVÁ

[a]*Department of Organic Technology, Faculty of Chemical Technology,*
Slovak Technical University, CS-812 37 Bratislava

[b]*Department of Environmental Chemistry and Technology, Faculty of Chemical Technology,*
Slovak Technical University, CS-812 37 Bratislava

The availability of a simple discrete statistical model for describing transient behaviour of the activated sludge process is studied in this work. Two-step changes of substrate feed concentration using laboratory treatment plant data are examined. The model sensibility to the number of regression parameters is evaluated. It is shown that the model represents a good approximation of the behaviour of an activated sludge process for the process control purposes.

In the previous papers we presented the results of dynamic modelling of an activated sludge process using nonlinear [1] and linearized [2] kinetic models. From the practical point of view, like the linearized model, the statistical model could also be available for the activated sludge process control. The purpose of this work is to give information about some results we have obtained from the modelling transient behaviour of an activated sludge process using laboratory treatment plant data.

Berthouex et al. [3] used similar technique to study the data from the municipal wastewater treatment plant. *Hansen et al.* [4] evaluated the impact of operating conditions, waste contributions, and environmental factors on industrial wastewater treatment plant performance using this technique.

Theoretical

Mathematical model

Activated sludge processes are usually expressed as continuous nonlinear systems. By the linearization technique (for example using the Taylor series) one can obtain linearized continuous system. A discrete system can be obtained by time's discretization. Let us consider a system with one input u and one output y, which is described by the relationship

$$y^{(n)} + \alpha_1 y^{(n-1)} + \alpha_2 y^{(n-2)} + \ldots \alpha_n y = \beta_0 u^{(m)} + \beta_1 u^{(m-1)} + \ldots \beta_{m-1} u^{(1)} + \beta_m u \quad (1)$$

We make approximations

$$y_t^{(1)} = \frac{y_t - y_{t-\Delta t}}{\Delta t}$$

$$y_t^{(2)} = \frac{y_t^{(1)} - y_{t-\Delta t}^{(1)}}{\Delta t} = \frac{y_t - 2y_{t-\Delta t} + y_{t-2\Delta t}}{(\Delta t)^2} \quad (2)$$

and so on for $y_t^{(3)}$, $y_t^{(n)}$; $u_t^{(1)}$, $u_t^{(m)}$.

Substituting approximations of derivatives into eqn (1) after a little manipulating, we obtain

$$y_t = a_1 y_{t-\Delta t} + a_2 y_{t-2\Delta t} + \ldots a_n y_{t-n\Delta t} + b_0 u_t + b_1 u_{t-\Delta t} + \ldots b_m u_{t-m\Delta t} + a_0 \quad (3)$$

where a_0 is a discretization error parameter.

For example, let us consider a continuous system described by the equation

$$y^{(3)} + \alpha_1 y^{(2)} + \alpha_2 y^{(1)} + \alpha_3 y = \beta_0 u^{(2)} + \beta_1 u^{(1)} + \beta_2 u \quad (4)$$

and a discrete system

$$y_t = a_1 y_{t-\Delta t} + a_2 y_{t-2\Delta t} + a_3 y_{t-3\Delta t} + b_0 u_t + b_1 u_{t-\Delta t} + b_2 u_{t-2\Delta t} + a_0 \quad (5)$$

There are following relationships

$$a_1 = \left(\frac{3}{\Delta t^3} + \alpha_1 \frac{2}{\Delta t^2} + \alpha_2 \frac{1}{\Delta t} \right) \frac{1}{d}$$

$$a_2 = \left(-\frac{3}{\Delta t^3} - \alpha_1 \frac{1}{\Delta t^2} \right) \frac{1}{d}$$

$$a_3 = \frac{1}{\Delta t^3} \frac{1}{d}$$

$$b_0 = \left(\beta_0 \frac{1}{\Delta t^2} + \beta_1 \frac{1}{\Delta t} + \beta_2 \right) \frac{1}{d} \quad (6)$$

$$b_1 = \left(\beta_0 \frac{2}{\Delta t^2} - \beta_1 \frac{1}{\Delta t} \right) \frac{1}{d}$$

$$b_2 = \beta_0 \frac{1}{\Delta t^2} \frac{1}{d}$$

and

$$d = \frac{1}{\Delta t^3} + \alpha_1 \frac{1}{\Delta t^2} + \alpha_2 \frac{1}{\Delta t} + \alpha_3 \quad (7)$$

One can discretize a continuous system using other approximations of derivatives, for example

$$y^{(1)} = \frac{y_{t+\Delta t} - y_t}{\Delta t}$$

$$y^{(2)} = \frac{y_{t+2\Delta t} - 2y_{t+\Delta t} + y_t}{\Delta t^2}$$

(8)

In this case, expressions for calculating a_i, b_j differ from eqn (6).

Without regarding the discretization procedure the final effect is that the continuous system is quite well described by the relationship (3), which can be straightforwardly used in an activated sludge process control. The best results are obtained when parameters a_0, a_n; b_0, b_m are estimated using the so-called recursive identification method [5]. Parameters a_i, b_j have statistical character, therefore the model expressed by eqn (3) is usually called the statistical model.

The model expressed by eqn (3) is also called the lagged regression model because the variables on the right-hand side are current values from the previous time "lags" For example, in our case today's effluent COD or NH_4^+ concentrations can be predicted using values of these variables from yesterday and some days ago, in the influent and effluent.

The parameters a_0, a_n; b_0, ... b_m have been estimated using linear regression [6] and the close set data (not recursive identification).

Results and discussion

The experiments were carried out using the apparatus of an activated sludge process described in [1]. The wastewater containing basic components of an industrial wastewater was used.

Two shock changes of substrate concentration of the feed were performed. The COD, NH_4^+, and hexamethylenetetramine (HMT) concentrations system responses were studied. The COD and NH_4^+ concentrations were determined by standard methods [7]. The HMT concentration was determined spectrophotometrically using the chromotropic acid reagent after acid hydrolysis of sample. The average value of COD concentration in the feed stream before the first shock was 3.896 kg m^{-3} The HMT and NH_4^+ feed concentrations were maintained as the average values of 0.48 kg m^{-3} and 0.15 kg m^{-3}, respectively.

Once having obtained a steady-state system behaviour the twofold decrease of substrate concentration followed. The hydraulic retention time, 24 h, and the solid retention time, 10 days, were maintained during experiments. The COD

and NH_4^+ concentration responses to this step change are presented in Figs. 1 and 2, respectively. An expressive NH_4^+ concentration system response was observed. The average NH_4^+ effluent concentration value of 0.106 kg m^{-3} was evaluated before the step change. The corresponding value in the new steady state (26 d after step change) was 0.0284 kg m^{-3} Practically no changes of HMT concentration were observed during this period of experiment.
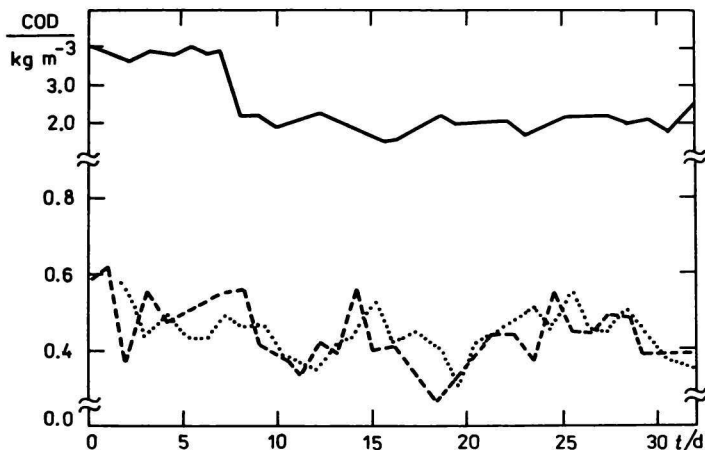

Fig. 1. COD concentration profiles after twofold decreasing of feed substrate concentration. ——— Feed substrate concentration, – – – – experimental effluent concentration, ········ calculated effluent concentration.
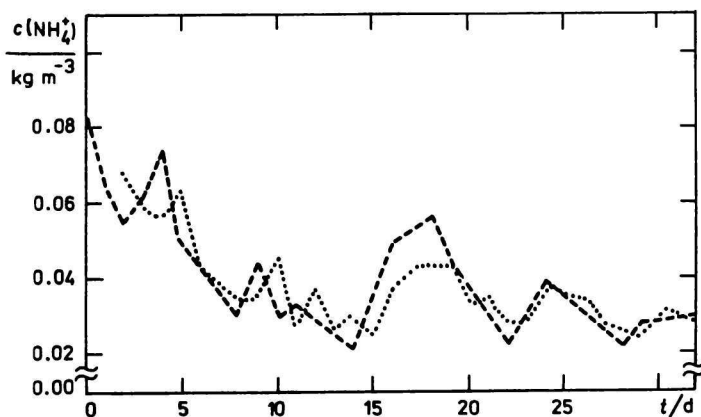

Fig. 2. NH_4^+ concentration profiles after twofold decreasing of feed substrate concentration. – – – – Experimental effluent concentration, calculated effluent concentration.

316

Fig. 3. COD concentration profiles after twofold increasing of feed substrate concentration. —————— Feed substrate concentration, ————— experimental effluent concentration, ·········· calculated effluent concentration.

Fig. 4. NH_4^+ concentration profiles after twofold increasing of feed substrate concentration. ————— Experimental effluent concentration, calculated effluent concentration.

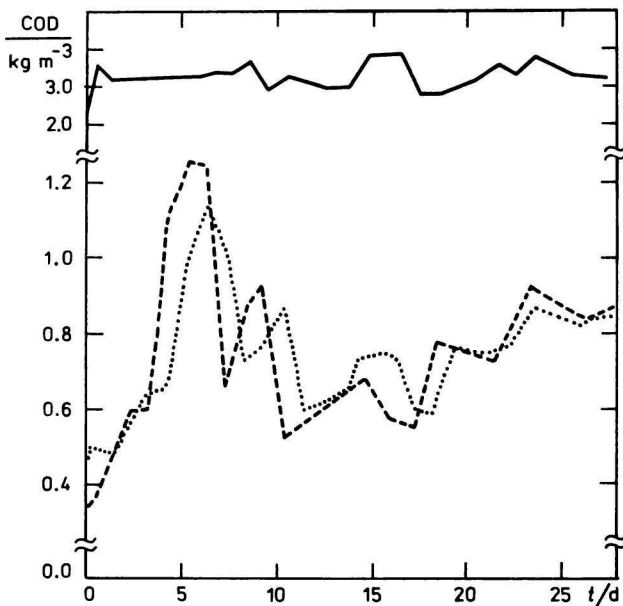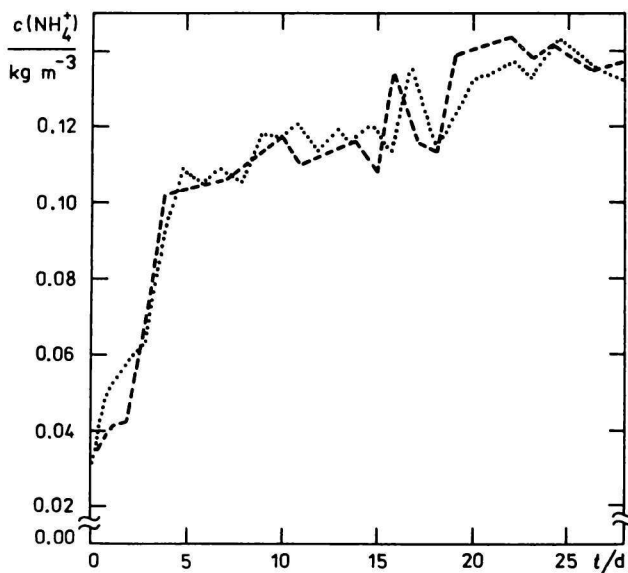After a new steady state had been obtained, the twofold step increase of the feed substrate concentration was performed. Figs. 3 and 4 show the COD and NH_4^+ concentration responses to this step change. A more expressive COD concentration response was observed. The significant decrease of COD efficiency removal during the first stage of the experiment is obvious from Fig. 3. Nearly the instantaneous NH_4^+ concentration system response to the shock load was observed. The average NH_4^+ concentration value in a new steady state (18 d after imposed shock) was 0.139 kg m^{-3} For comparison, the steady-state average effluent COD concentration values before the first and after the second step change were 0.642 kg m^{-3} and 0.789 kg m^{-3}, respectively. A significant decrease of an effluent HMT concentration was observed at this stage of experiment. Due to expressive decrease of pH value (5.7—6.2) at this stage the efficiency of HMT removal reached the value of 53 %. In less than 5 d after the step shock was employed, pH values increased almost to the value of 7.0 and effluent HMT concentrations approached the influent HMT values. In less than 10 d after the step change was carried out the macroscopic activated sludge changes were observed. After a new steady state was reached, the colour of activated sludge changed to the original one, but no changes in flocs size occurred.

Figs. 1 to 4 also show calculated COD and NH_4^+ concentrations profiles applying the statistical model expressed by eqn (3).

The sensitivity on regression parameters number was examined. The values of squared deviations of experimental and calculated values obtained by selecting a different number of regression parameters are summarized in Tables 1 and 2 for COD and NH_4^+ concentration responses evaluation, respectively. From Tables 1 and 2 it is evident that the significant decreasing of weighted average

Table 1

The sum of squared deviations between the COD concentration experimental and calculated values for a different number of parameters

	$\omega \cdot 10$		
	$m = 4$	$m = 5$	$m = 6$
3	0.1452	0.1417	0.1300
4	0.1435	0.1385	0.1300
5	0.1326	0.1268	0.1213
6	0.1327	0.1225	0.1177
7	0.1316	0.1316	0.1177
8	0.1316	0.1217	0.1164
9	0.1316	0.1225	0.1164

Table 2

The sum of squared deviations between the NH_4^+ concentration experimental and calculated values for a different number of parameters

	$\omega \cdot 10^4$		
	$m = 4$	$m = 5$	$m = 6$
3	0.9865	0.9562	0.9456
4	0.9452	0.9277	0.9224
5	0.8815	0.8704	0.8756
6	0.8815	0.8704	0.8756
7	0.8815	0.8704	0.8756
8	0.8815	0.8754	0.8756
9	0.8791	0.8744	0.8836

of the squared deviations value was obtained by selecting the number of parameters a_i and b_j of eqn (*3*) equal to 5. The increased number of regression parameters did not contribute significantly to the decrease of this value. Based on the Student test the number of statistically significant regression parameters did not exceed five for a_i and b_j in both COD and NH_4^+ concentration profiles evaluation.

Conclusion

Our results confirmed that the discrete statistical model is suitable for describing activated sludge process dynamics. From the figures presented in this paper there is evident a reasonably good fit between the calculated results and experimental data. A little drift between experimental and computational responses is caused by considering the studied system as a system with one input and one output. Better mathematical description of the system and removal of the response drift are expected by considering more inputs and outputs. The computational time for model parameters estimation was significantly shorter in comparison to nonlinear (100 times shorter) or linearized (5 times shorter) models. Therefore, we concluded that the discrete statistical model is the best approximation of real activated sludge process behaviour for process control purposes. In practical applications of this model one must pay attention to flow and concentration (in some cases also temperature) intervals for model validity. Extrapolations are not convenient.

Symbols

a_i, b_j parameters of the discrete statistical model
c_j concentration of j-th component [kg m^{-3}]
COD   chemical oxygen demand
n number of parameters a_i
m number of parameters b_j
t time [d]
u input
$u^{(i)}$ i-th time derivative of u
u_t value of u in time t
y output
$y^{(j)}$ j-th time derivative of y
y_t value of y in time t
α_i, β_j parameters of the continuous model
Δt time interval
ω sum of squared deviations between experimental and calculated data

References

1. Derco, J., Králik, M., Hutňan, M., and Gubová, P., *Chem. Papers 44*, 409 (1990).
2. Derco, J., Králik, M., Gubová, P., and Bodík, I., *Chem. Papers 44*, 659 (1990).
3. Berthouex, P. M. *et al., Water Res. 10*, 689 (1976).
4. Hansen, J. L., Fiok, A. E., and Hovious, J. C., *J. Water Pollut. Contr. Fed. 52*, 1966 (1980).
5. Mikleš, J. and Hutla, V., *Teória automatického riadenia.* (Theory of the Automatic Control.) Alfa Publishers, Bratislava, 1986.
6. Rektorys, K. *et al., Přehled užité matematiky.* (Survey of Applied Mathematics.) Nakladatelství technické literatury (Publishers of Technical Literature), Prague, 1981.
7. Horáková, M., Lischke, P., Pekárková, K., and Grünwald, A., *Metody chemické analýzy vod.* (Methods of Chemical Analysis of Waters.) Nakladatelství technické literatury (Publishers of Technical Literature), Prague, 1981.

Translated by J. Derco