Rotary disc extractor simulation

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Residence time distribution, obtained experimentally, has been compared with that calculated from single-parameter stage models and from a one-parameter dispersion model. Parameters of models have been determined by the Fibonacci method. For the studied experimental conditions, rotary disc extractor was best simulated by a cascade of ten perfectly mixed vessels with backflow.

Экспериментальным путем установленные распределения времени пребывания сравнивались с рассчитанными на основании однопараметрических ступенчатых моделей и однопараметрической дисперзной модели. Параметры моделей определялись методом Фибоначи. Условия экспериментов в ротационно-дисковом экстракторе наилучшим образом симулировались каскадом 10-ти идеальных смесителей (аппаратов с совершенным перемешиванием) с возвратным потоком.

In simulating liquid flow in a RD extractor, *Strand* [1], *Stemerding* [2], *Míšek* [3, 4], *Westerterp* [5], and others have all based themselves on the conception of piston liquid flow through an axial dispersion device (dispersion model). The idea of stage flow through the equipment was taken up by *Stainhorp* [6], *Míšek* [3], and others.

The present paper deals with the application of a one-parameter dispersion model and of single-parameter stage models to one-phase flow in a RD extractor.

Dispersion model

For a dispersion model bounded from two sides and for the initial condition given by a unit function, *Yagi* [7] has derived a relation from which, by derivation, we can get function E in the form

$$E(\Theta)=2\exp\left(\frac{Pe}{2}\right)\sum_{i=1}^{\infty}\frac{(-1)^{i+1}\alpha_i^2}{\alpha_i^2+\left(\frac{Pe}{2}\right)^2+Pe}\exp\left[-\Theta\frac{\alpha_i^2+\left(\frac{Pe}{2}\right)^2}{Pe}\right] \tag{1}$$

where α_i are positive roots of transcendental equations

$$\frac{\alpha_i}{2}\operatorname{tg}\frac{\alpha_i}{2}=\frac{Pe}{4} \qquad i=1, 3, 5, \ldots \tag{1a}$$

$$\frac{\alpha_i}{2}\operatorname{cotg}\frac{\alpha_i}{2}=-\frac{Pe}{4} \qquad i=2, 4, 6, \ldots \tag{1b}$$

Cascade of ideal mixers (CIM)

To calculate function E of a cascade of N ideal mixers of equal volume, *Levenspiel* [8] has proposed to use the following relationship

$$E(\Theta)=\frac{N^N}{(N-1)!}\,\Theta^{N-1}\exp(-N\Theta) \tag{2}$$

In a cascade of ideal mixers with a noninteger number of terms N it is possible to calculate function E using the relation [9]

$$E(\Theta)=\frac{N^N}{\Gamma(N)}\,\Theta^{N-1}\exp(-N\Theta) \tag{2a}$$

where $\Gamma(N)$ is function gamma.

Cascade of ideal mixers with backflow

The model shown in Fig. 1 is a two-parameter model with parameters N and β. Coefficient β is the ratio of the backflow to the inflow into the equipment. It was assumed that the value of β remains the same, *i.e.* that it does not depend on the

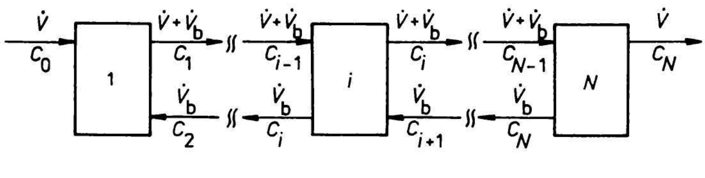

Fig. 1. Cascade of ideal mixers with backflow.

serial number of the mixer. The hold-up of liquid between the mixers tends to zero in limit. Backflow CIM is described by material balance equations (in unstable state) on a tracer [6]

$$\frac{1}{N}\frac{\mathrm{d}c_i}{\mathrm{d}\Theta}=c_{i-1}-(1+\beta)\,c_i+\beta c_{i+1} \qquad i=1$$

$$\frac{1}{N}\frac{\mathrm{d}c_i}{\mathrm{d}\Theta}=(1+\beta)\,c_{i-1}-(1+2\beta)\,c_i+\beta c_{i+1} \qquad 1<i<N \tag{3}$$

$$\frac{1}{N}\frac{\mathrm{d}c_i}{\mathrm{d}\Theta}=(1+\beta)\,c_{i-1}-(1+\beta)\,c_i \qquad i=N$$

For the initial condition given by Dirac δ function, *Roemer* and *Durbin* [10] obtained the following relationship for function E

$$E(\Theta)=\sum_{i=1}^{N} A_i \exp\,(s_i\Theta) \tag{4}$$

where

$$s_i=\frac{N}{1-\gamma}\,[2\sqrt{\gamma}\cos \varkappa_i-(1+\gamma)] \tag{4a}$$

$$A_i=-2N\gamma^{-\frac{N}{2}}\frac{\sin^2 \varkappa_i}{D'(\varkappa_i)} \tag{4b}$$

$$\gamma=\frac{\beta}{1+\beta} \tag{4c}$$

and $\varkappa_i$ are nonzero roots of the transcendental equation

$$D(\varkappa)=\gamma^{-\frac{1}{2}}\sin\,[(N+1)\,\varkappa_i]-2\sin\,(N\,\varkappa_i)+\sin\,[(N-1)\,\varkappa_i] \tag{4d}$$

from the interval $\langle 0,\pi \rangle$. Quantity $D'(\varkappa)$ denotes the derivative of $D(\varkappa)$ in (*4d*).

Experimental

Measurement was done on the equipment shown in Fig. 2. The rotary disc extractor, 0.8 m long, was 0.05 m in diameter and had 10 mixing compartments. The over-all working part of the extraction column measured 0.3 m. Water from the mains was led from above by an inlet pipe emptying into the column at about 0.01 m above water-level. Stator rings were of 0.035 m in internal diameter and stator discs of 0.03 m diameter each. The eleventh (last) stator ring was a solid one with only a hole in it for the axis and water outlet and had an air-gap beneath it.

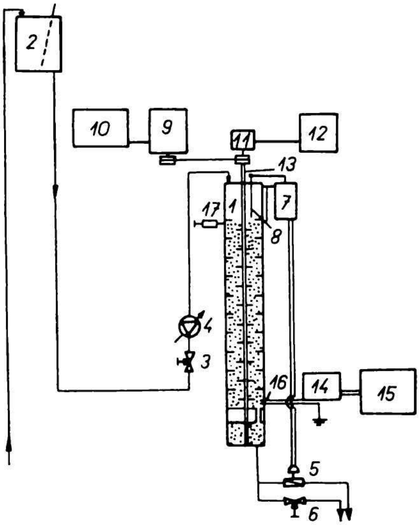

Fig. 2. Diagram of experimental equipment.
1. RD Extractor; *2.* water storage tank; *3.* regulating valve; *4.* rotameter; *5.* solenoid valve; *6.* regulating valve; *7.* electronic relay; *8.* electronic relay automatic control system; *9.* electric motor with adjustable number of revolutions; *10.* electric motor regulating box; *11.* tachometer dynamo; *12.* revolution indicator; *13.* RD extractor axis; *14.* conductivity measuring instrument; *15.* recording instrument; *16.* conductivity sensor; *17.* tracer spraying.

Such arrangement made it possible to realize boundary conditions in eqns (*1—3*) or (*4*). Curves E have been experimentally obtained by the impulse-response method. As tracer we have used KCl solution of $w = 0.200$ composition. Two cm^3 of KCl solution were injected into the first mixing compartment. The response was measured with a continual conductivity recording device. Platinum wires 20 mm long and 1 mm in diameter were inserted into the tenth mixing compartment. Throughout measuring, the temperature in the column was kept constant with accuracy of ± 0.1°C.

Results and discussion

To calculate optimum parameters of the stated models, we have chosen, as accordance criterion, minimization function

$$U(p) = \sum_{i=1}^{n} [E_{ei}(\Theta) - E_{ci}(\Theta)]^2 = \min \tag{5}$$

The closeness of the experimentally obtained and the calculated function E has been judged by the regression coefficient

$$R_c = \frac{n \sum_{i=1}^{n} (E_{ei} E_{ci}) - \sum_{i=1}^{n} E_{ei} \sum_{i=1}^{n} E_{ci}}{\sqrt{\left[n \sum_{i=1}^{n} E_{ei}^2 - \left(\sum_{i=1}^{n} E_{ei}\right)^2\right]\left[n \sum_{i=1}^{n} E_{ci}^2 - \left(\sum_{i=1}^{n} E_{ci}\right)^2\right]}} \tag{6}$$

In calculating the roots of transcendental eqns (*1a*), (*1b*), and (*4c*) we have used the numerical method of halving the interval. The accuracy of iterative calculations was taken to be $\varepsilon = 1 \times 10^{-5}$. For calculating the sum in eqn (*1*) we have chosen such a number of terms i for which it holds that $i+1$ is less than the selected accuracy value of $\varepsilon = 1 \times 10^{-5}$ (maximum number of terms was 17).

In calculating optimal N in relationship (*2*) we set out from the integer value of N_0 which was certainly lower than that of optimal N.

$$N_0 = \frac{1}{\sigma^2} - 5 \tag{7}$$

where

$$\sigma^2 = \frac{\sum_{i=1}^{n} \Theta_i^2 E_{ei}}{\sum_{i=1}^{n} E_{ei}} - 1 \tag{8}$$

If N_0 was less than 2 we took it to be 2. Successively increasing N_0 by one we determined the minimum value of relationship (*5*) for N_{opt}.

In calculating optimal N in relationship (*2a*) we have fixed the values $N_{opt} - 2$ and $N_{opt} + 2$ to be the boundaries for the Fibonacci method.

Function E of the CIM with backflow has been calculated both from relationships (*4, 4a—d*) and from the numerical solution of the system of equations (*3*) by the Runge—Kutta method modified by Merson. The initial condition of Dirac δ function was simulated as unit impulse ($\Theta = 0.001$ and $c = 1000$) of short duration.

Table 1

Calculated parameters of models

Experiment *I*	CIM according to (*2*)		CIM according to (*2a*)		CIM with backflow		Diffusion model	
	N	R_c	N	R_c	γ	R_c	Pe	R_c
1	5	0.9891	4.78	0.9972	0.43	0.9993	7.48	0.9967
2	4	0.9897	4.20	0.9941	0.50	0.9972	6.48	0.9956
3	3	0.9465	2.72	0.9796	0.71	0.9981	3.24	0.9980
4	5	0.9713	4.43	0.9875	0.48	0.9979	6.73	0.9980
5	3	0.9722	3.17	0.9843	0.65	0.9997	4.24	0.9990
6	3	0.9811	3.26	0.9891	0.62	0.9927	4.49	0.9912
7	3	0.9398	2.63	0.9800	0.72	0.9996	3.19	0.9996
8	4	0.9821	4.30	0.9893	0.50	0.9996	6.45	0.9995
9	4	0.9745	3.89	0.9867	0.55	0.9993	5.82	0.9983

Both methods provided almost identical values of function E. Initial values γ for the Fibonacci method of optimalization were taken to be 0.1 or 0.9. All the calculations of function E for the CIM with backflow have been done for $N = 10$. The calculated values of parameters of individual models are given in Table 1. On the basis of the regression coefficient and of the graphical representation (Figs. 3 and 4) the highest degree of conformity has been attained between experimentally

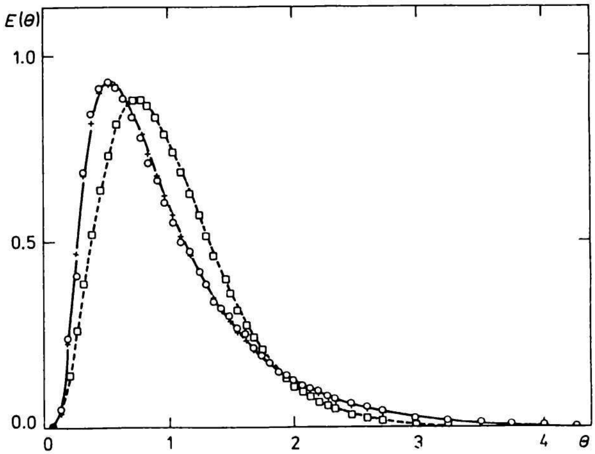

Fig. 3. Measured and calculated curve E compared (for conditions of experiment 3). ○ Experimental values; + dispersion model; □ CIM – relationship (*2*).

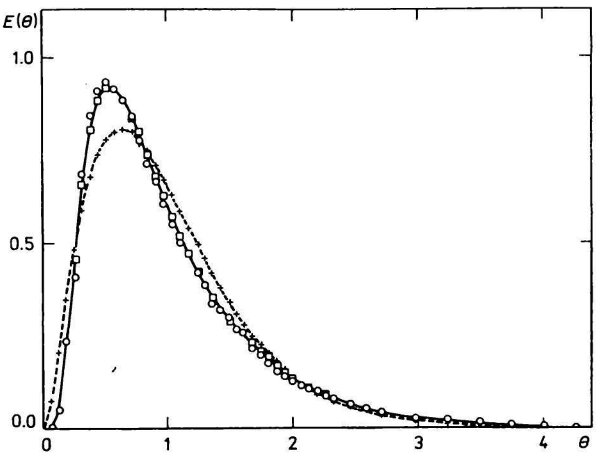

Fig. 4. Measured and calculated curve E compared (for conditions of experiment 3). ○ Experimental values; + CIM — relationship (*2a*); □ CIM with backflow.

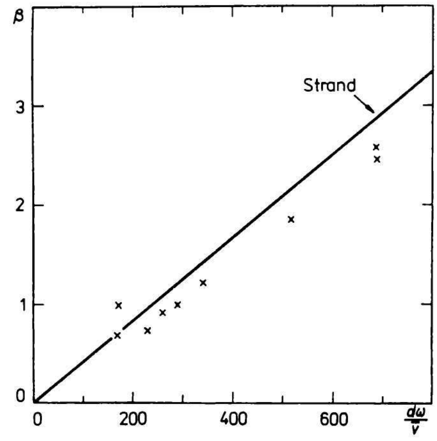

Fig. 5. Dependence of β on $D\omega/\bar{v}$.

obtained function E_e and the calculated function E_c for the CIM model with backflow. The values of coefficients β correspond to values β given by *Strand* correlation [1] (see Fig. 5 and Table 2)

$$\beta = 0.09\,\frac{D\omega}{\bar{v}}\left(\frac{D}{D_c}\right)^2\left[\left(\frac{O_s}{D_c}\right)^2 - \left(\frac{D}{D_c}\right)^2\right] \tag{9}$$

which may be obtained from the original correlation using the relationship

$$\frac{1}{Pe'} = 0.5 + \beta \tag{10}$$

The dispersion model has shown but a slightly lower conformity between the measured and the calculated values of function E than that presented by a CIM with backflow. The latter, however, gives better results for a noninteger than for an integer number of N.

Table 2

Experimental conditions and results obtained

I	$\dot{V}/\mathrm{cm^3\,s^{-1}}$	$\omega/\mathrm{s^{-1}}$	$D\omega/\bar{v}$	β
1	3.33	10	172	0.75
2	5.00	15	172	1.00
3	2.50	30	687	2.45
4	3.33	15	257	0.92
5	3.33	30	515	1.85
6	2.50	30	687	2.57
7	5.00	25	287	1.00
8	5.00	30	343	1.22
9	4.17	20	229	0.75

Symbols

A	coefficient defined by eqn (*4b*)
C	tracer concentration
c	dimensionless concentration
$D(\varkappa)$	function defined by eqn (*4d*)
D	stator disc diameter
D_c	column diameter
$E(\Theta)$	residence time distribution function
L	length of working part of the extractor
n	number of experimental values
N	number of stages in CIM
O_s	stator hole diameter
$Pe=\dfrac{\bar{v}L}{\varepsilon}$	Peclet criterion
$Pe'=\dfrac{Pe}{N}$	Peclet criterion
p	parameter
R_c	regression coefficient
s	coefficient defined by eqn (*4a*)
t	time
$\dot{V}$	volume flow
V	extractor capacity (volume)
$\bar{v}$	average velocity
w	mass fraction
β	backflow coefficient $\dot{V}_b/\dot{V}$
ε	turbulent diffusion coefficient
γ	coefficient defined by eqn (*4c*)
Γ	function gamma
$\delta(\Theta)$	Dirac δ function
$\Theta=\dfrac{t\dot{V}}{V}$	dimensionless time
σ^2	variance of the residence time distribution
ω	revolution frequency

Indices

e	experimental
b	backflow
c	calculated

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